AE-801

M.A./M.Sc. (Previous)
Term End Examination, 2016-17

MATHEMATICS

Paper - III

Topology

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. All questions carry equal marks.

1. (*a*) Let B and B' be bases for topology T and T' respectively on X. Then prove that the following statements are equivalent :

(*i*) T' is finer than T.

(*ii*) For each $x \in X$ and each basis element $\beta \in B$ containing x, there is a basis element $\beta' \in B$ such that $x \in \beta' \subset \beta$.

(*b*) If B is a basis for topology of X, then prove that the collection $B_y = \{\beta \cap y : \beta \in B\}$ is a basis for the subspace topology on y.

2. (*a*) If A is a subset of the topological space X and let A' be the set of all limit points of A, then prove that $\overline{A} = A \cup A'$.

(*b*) State and prove pasting lemma.

3. (*a*) Prove that every second countable space is first countable but converse is not true in general.

(*b*) Prove that image of a connected space under a continuous map is connected.

4. (*a*) If A is a connected subspace of X and $A \subset B \subset \overline{A}$, then prove that B is also connected.

(*b*) Prove that a finite Cartesian product of connected spaces is connected.

5. (*a*) Prove that a relation on a topological space defined by "$x \sim y$" $\rightarrow$ "If there is a path from x to y" is an equivalence relation.

(*b*) Prove that a space X is locally connected if and only if for every open set U of X each component of U is open in X.

6. (*a*) Prove that every closed subspace of a compact space is compact.

(*b*) Prove that every compact space is limit point compact but its converse is not true in general.

7. (*a*) Prove that every compact subspace of a Hausdorff space is closed.

(*b*) Prove that image of a compact space under a continuous map is compact.

8. (*a*) Show that every compact Hausdorff space is normal.

(*b*) Prove that product of regular space is regular.

9. (*a*) Show that every regular space with countable basis is normal.

(*b*) If X is a Hausdorff space, then show that a net in X converges to at most one point.

10. State and prove Urysohn's metrization theorem.

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