AE-800

M.A./M.Sc. (Previous)
Term End Examination, 2016-17

MATHEMATICS

Compulsory

Paper - II

Real Analysis and Measure Theory

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. All questions carry equal marks.

1. (*a*) State and prove the fundamental theorem of calculus.

(*b*) Lef f be a bounded function and α be a monotonically increasing function on $[a, b]$. Then $f \in R(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition P such that

$$U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$$

Prove it.

2. (*a*) Let f be a continuous function defined on $[a, b]$. Then prove that there exists a sequence of polynomials which converges uniformly to f on $[a, b]$.

(*b*) State and prove Tauber's theorem on power series.

3. (*a*) Show that the series

$$1 - x + x^2 - x^3 + \ldots. \ (0 \le x \le 1)$$

admits of integration term-by-term in $0 \le x \le 1$ although it is not uniformly convergent in $[0, 1]$.

(*b*) Show that

$$\int_a^b f \, dx = \lim_{n \to \infty} \int_a^b fn \, dx$$

4. State and prove the inverse function theorem.

5. (*a*) Show that the power series $1 + 2x + 3x^2 + 4x^3 + \ldots.$ has radius of convergence equal to 1.

(*b*) If a power series $\Sigma a_n x^n$ diverges for $x = x_0$, then prove that it diverges for every $x = x_1$ when $|x_1| < |x_0|$.

6. (*a*) Find the radius of convergence of the series

$$\frac{1}{2}x+\frac{1.3}{2.5}x^2+\frac{1.3.5}{2.5.8}x^3+.....$$

(*b*) If the series $\sum_{n=0}^{\infty} a_n$ is convergent and has the sum S, then prove that the series $\sum_{n=0}^{\infty} a_n x^n$ is uniformly convergent for $0 \le x \le 1$ and $\lim_{x\to 1} \sum_{n=0}^{\infty} a_n x^n = S$.

7. (*a*) Prove that the intersection of a finite number of measurable sets is measurable.

(*b*) Prove that the interval (a, ∞) is measurable.

8. (*a*) Show that a Borel measurable set is Lebesgue measurable.

(*b*) If E_1 and E_2 are any measurable sets, then prove that

$$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2).$$

9. (*a*) Find the rectangular parallelopiped of surface area a^2 and maximum volume.

(*b*) Let f be a bounded function defined on $[a, b]$. If f is Riemann integrable on $[a, b]$. Then prove that it is Lebesgue integrable on $[a, b]$ and

$$R\int_a^b f(x)dx = \int_a^b f(x)dx$$

10. (*a*) Let ϕ be a convex function on $(-\infty, \infty)$ and f an integrable function on $[0, 1]$. Then $\int_0^1 \phi(f(t))\,dt \geq \phi\left[\int_0^1 f(t)dt\right]$. Prove it.

(*b*) State and prove Holder's Inequality.

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