AE-803

M.A./M.Sc. (Previous)
Term End Examination, 2016-17

MATHEMATICS

Optional

Paper - II

Advanced Discrete Mathematics

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. Answer to each question should begin on a fresh page. All questions carry equal marks.

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1. (*a*) Show that $p \vee \sim (q \wedge r)$ is equivalent to $(p \vee \sim q) \vee \sim r$.

(*b*) State and prove De-Morgan law.

2. (*a*) Show that the following argument is valid :

$$\begin{array}{c} p \\ p \Rightarrow q \\ \hline q \end{array}$$

(*b*) Explain quantifiers in detail.

3. (*a*) Define semigroup and monoid.

(*b*) Show that every finite semigroup has an idempotent element.

4. (*a*) Show that for any commutative monoid $(M, *)$, the set of idempotent elements of M forms a submonoid.

(*b*) Let $(M, *)$, be a monoid, then show that there exists a subset $T \subseteq M^M$ such that $(M, *)$ is isomorphic to monoid $(T, *)$.

5. (*a*) Let $(L, \leq)$ be a lattice in which * and $\oplus$ denote the operations of meet and join respectively. Then show that for any $a, b \in L$

$$a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$$

(*b*) Show that every chain is a distributive lattice.

6. (*a*) State and prove associative law in Boolean algebra.

(*b*) Show that complement element of every element is unique in Boolean algebra.

7. (*a*) Show that the value of complete disjunctive normal form is one.

(*b*) In any Boolean algebra, show that

$$(a + b)\ (b + c)\ (c + a) = a.b + b.c + c.a$$

8. (*a*) Explain finite state machine in detail.

(*b*) Show that $L = \{a^k : k = i^2,\ i \geq 1\}$ is not a finite state language.

9. (*a*) Explain the following terms :

(*i*) Path

(*ii*) Circuit

(*b*) Show that a tree with *n* vertices has *n*–1 edges.

10. (*a*) State and prove Euler formula for connected graph.

(*b*) What do you mean by Konigsberg Bridge problem ?