AE-806

M.A./M.Sc. (Final)
Term End Examination, 2016-17

MATHEMATICS

Compulsory

Paper - II

Partial Differential Equations,
Mechanics and Gravitation

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. All questions carry equal marks.

1. (*a*) (*i*) Find $L^{-1}\left\{\frac{p+2}{p^2-4p+13}\right\}$

(*ii*) Solve the differential equation

$$\frac{\partial^2 y}{\partial x^2} - \frac{\partial^2 y}{\partial t^2} = xt, \text{ where}$$

$$y = 0 = \frac{\partial y}{\partial t} \text{ at } t = 0 \text{ and } y(0, t) = 0$$

by using Laplace Transform.

(*b*) Find the Fourier transform of $f(x)$ defined by

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$ and hence evaluate

(*i*) $$\int_{-\infty}^{\infty} \frac{\sin pa \cos px}{p} \, dp$$

(*ii*) $$\int_{0}^{\infty} \frac{\sin p}{p} \, dp$$

2. (*a*) (*i*) Solve the Differential Equation

$$(t+y+z)\frac{\partial t}{\partial x} + (t+x+z)\frac{\partial t}{dy} + (t+x+y)\frac{\partial t}{dz} = x+y+z$$

(*ii*) Classify the following equation :

$$(1-x^2)\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + (1-y^2)\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + 3x^2 y\frac{\partial z}{\partial y} - 2 = 0$$

(*b*) Show that the Green's function $G(r, r')$ has the symmetric property.

3. (*a*) Define the Harmonic function. If a harmonic function vanishes everywhere on the boundary, then show that it is identically zero everywhere.

(*b*) Derive the one-dimensional wave equation.

4. (*a*) Let u be a harmonic function in the interior of a rectangle $0 \le x \le a$, $0 \le y \le b$ in xy plane satisfying Laplace equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

with $u(0, y) = 0, \quad u(a, y) = 0,$
$u(x, b) = 0, \quad u(x, 0) = f(x)$

Determine u for above problem, by using the method of separation of variables.

(*b*) Find the fundamental solution of Heat equation.

5. (*a*) Derive Routh's Equations.

(*b*) Give the definition of cyclic coordinates and show that the generalised momentum conjugate to a cyclic coordinate is conserved.

6. (*a*) Show that Poisson's Brackets are invariant under canonical transformation.

(*b*) Derive Hamilton-Jacobi Equation.

7. (*a*) State and prove Donkin's Theorem.

(*b*) Define the following :

(*i*) Generalised coordinates

(*ii*) Poisson Brackets

(*iii*) Hamiltonian

8. (*a*) Solve the Brachistochrone problem.

(*b*) Show that the transformation defined by

$$q=\sqrt{[2P]}\sin Q$$

$$p=\sqrt{[2P]}\cos Q$$

is canonical.

9. (*a*) Find the attraction of thin uniform spherical shell at an external and internal point.

(*b*) Show that the potential of a uniform circular disc, of mass M and radius a, at a point in its plane distant c from its centre, is

$$\frac{4\gamma M}{\pi a^2}\int_0^{\frac{\pi}{2}}\sqrt{a^2-c^2\sin^2\theta}\ d\theta \quad \text{or}$$

$$\frac{4\gamma M}{\pi a^2}\int_0^{\sin^{-1}\frac{a}{c}}\sqrt{a^2-c^2\sin^2\theta}\ d\theta$$

according as c is less or greater than a.

10. (*a*) Derive the Poisson Equation.

(*b*) Find the distribution of matter which will produce the following potentials :

V = 1 within the ellipsoid

$$\frac{x^2}{\mu^2 a^2}+\frac{y^2}{\mu^2 b^2}+\frac{z^2}{\mu^2 c^2}=1;\ (\mu<1)$$

$$V=\frac{1}{1-\mu^2}\left[1-\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}\right]$$

between the above ellipsoid and

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1;$$

$V=0$ outside the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$$

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