PC - 481

(514) M.A./M.Sc. MATHEMATICS (FOURTH SEMESTER)

Examination JUNE 2020

Compulsory/Optional

Group-

Paper-II

PARTIAL DIFFERENTIAL EQUATIONS, MECHANICS AND GRAVITATION-II

Time:- Three Hours | Maximum Marks: 080

Minimum Passing Mark- 29

नोट : दोनो खण्डों निर्देशानुसार उत्तर दीजिए। प्रश्नों के अंक उनके दाहिनी ओर अंकित हैं।

Note: Answer from Both the Section as Directed. The Figures in the right-hand margin indicate marks.

Section - A

1. Attempt all question: 1X10

(a) Define constraints.

(b) Define Non-Holominic constraints.

(c) Define Generalised Velocity and Generalised Acceleration.

(d) Define Hamiltonian.

(e) What are canonical Transformations?

(f) Define equipotential surfaces.

(g) Write expression for potential of infinite rod (from both sides)

(h) Write the formula for potential of solid sphere.

(i) What is the potential of spherical shell of finite thickness?

(j) Define surface Density.

2. Answer the following question : 2X5

(a) What is generalised force.

(b) Define Lagrange's Equation of First-Kind.

(c) Find the differential Equation of simple pendulum of length .

(d) Define potential and find potential of a body of man M at a point P.

(e) Find the potential if law of attraction in the inversely proportional to the n^{th} power of the distance.

खण्ड/ Section - B

Answer all questions : 12X5

3. (a) Find the shortest distance between two points in a plane.

(b) Show that the transformation

$$P = \frac{1}{2}(p^2 + q^2), Q = \tan^{-1}(\frac{q}{p})$$

is canonical (by using Hamilton's canonical Equation)

OR

(a) Show that the transformation

$$Q = \frac{1}{p}, P = qp^2 \text{ is canonical.}$$

(b) If $[p_l, q_i], [q_l, q_i]$ are Langranzian brackets and $[p_l, p_j], [q_l, p_j]$ are the poisons brackets, then prove that

$$\sum_{l=1}^{n}[p_l, q_i][p_l, p_j] + \sum_{l=1}^{n}[q_l, q_i]\,[q_l, p_j] = 0$$

4. (a) Find the relation between poison and Lagranze Brackets.

(b) Define the Lagrange's Equation of first Kind.

OR

Define the Hamilton's canonical equation.

5. (a) Define Lagrange's Brackets. Show that Lagrange's Brackets is invariant under canonical transformation.

(c) Show that Lagrange's brackets do not obey the commutative law.

OR

Define Poisson Bracket and By using it solve the problem .

For what values of m and n do the transformation equations

$$Q = q^m cosnp$$
$$P = q^m sinnp$$

6. Find the potential of their uniform rod of lenth a thickers k and density S at an external point.

OR

Find the potential of a thin uniform spherical shell at an external and internal points.

7. Show that the potential of an infinite uniform thin cylindrical shell at point p is

$$\text{C- } 4\pi ya\, M log\, a \quad or \quad C - 4\pi ya\, M log\, r$$

According a P is inside or outside the cylinder the mass per unit area being M, the radian of shell a, and r being the distance of P form the axis.

OR

If $Q(x, y, z)$ be the potential at an internal point $P(x, y, z)$ of a thin heterogeneous spherical shell, then the potential at an external point $P'(x', y', z')$ is

$$\frac{a}{r'}\left(\frac{a^2x'}{r'^2}, \frac{a^2y'}{r'^2}, \frac{a^2z'}{r'^2}\right)$$

Where a is the radian of the shell and r the distance of p from its centre.