PD-481-CV-19

M.A./M.Sc. (4th Semester)

Examination, June-2021

MATHEMATICS

Paper-II

PARTIAL DIFFERENTIAL EQUATIONS, MECHANICS & GRAVITATION-II

Time : Three Hours] **[Maximum Marks : 80**

[Minimum Pass Marks : 29

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Answer all the questions:- 1x10=10

(a) **Define Generalized velocity.**

(b) **Define Haniltouiam.**

(c) **Define cyclic coordinates.**

(d) **Write down Routh's equations.**

(e) **Define Poisson Bracket.**

(f) **Define Ragrange's Bracket.**

(g) **Write expression for potential of spherical shell of finite thickness.**

(h) **Write down the expression for potential of dise.**

(i) **Define Potential.**

(j) **Write down expression for potential of thin uniform rod.**

2. Answer the following questions:- 2x5=10

(a) **Define Lagrange's equation of second kind.**

(b) **Find the shortest-distance between two points in a plant.**

(c) **Define equipotential surfaces.**

(d) **If a curve passing through two end points (x_1, y_1) and (x_2, y_2) has been rotated about y axis then find the minimum surface of revolution.**

(e) **Find the differential equation of simple pendulum.**

Section-B

12x5=60

Answer the following questions:-

3. Define Ruthian function and Derive Rouths equation of motion.

OR

Derive Lagrange's equation of second Kinel.

4. Derive Hamiltons canonical equation.

OR

State and prove Jacobi-Poisson theorem.

5. A solid homogeneous cylinder of radius r rolls without slipping on the inside of a stationary large cylinder of radius R.
Than (a) find the equation of motion.
And (b) What is the period of small oscillations about the stable equilibrium position.

OR

For a certain canonical transformation it is known that

$$Q = \sqrt{(q^2 + p^2)}, \qquad F = \frac{1}{2}(q^2 + p^2)\, tan^{-1}\frac{q}{p} + \frac{1}{2}qp$$

Then find $p(q, p)$ and $F(q, Q)$

6. **Find the potential of solid sphere at an internal and external points.**

OR

Find the potential of circular dise at an external point on its axis.

7. **Show that the potential of a uniform spherical shell of small thickness K, density ρ and radius a at an external point distant c from the centre is**

$$\frac{2\pi\gamma kpa}{(n+1)(n+3)e}[(c+a)^{n+2}-(c-a)^{n+3}]$$

If the law of force be that of the n^{th} power of the distance.

OR

The values of potential v at any point at a distance r from a fixed point O are

$$V = 2\pi\gamma\rho(a^2-b^2) \qquad if\ r<b<a$$

$$V = 2\pi\gamma\rho\left(a^2-\frac{r^2}{3}-\frac{2b^3}{3r}\right) \qquad if\ b<r<a$$

and $$V = \frac{4\pi\gamma\rho}{3}\left(\frac{a^3-b^3}{r}\right) \qquad if\ b<a<r$$

Show that the attracting system is a spherical shell of density ρ, whose boundaries are spheres of centre σ and radii a & b.