PC – 480 CV-19

M.A./M.Sc. (Fourth Semester)

Examination June2020

MATHEMATICS

Compulsory Paper-I

Integration Theory and Functional Analysis-II

Time : Three Hours] [Maximum Marks : 80

[Minimum Pass Marks : 29

Note : Answer from **both** the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Fill in the blank of the following:- [1x10=10]

(i) Let N be a normed linear space and f(x) = 0, π f ∈ N* Then x = ...

(ii) The Norm of Identity bounded linear operator on a linear space N ≠ {0} is ....

(iii) Normed linear space is separable if it's .... is separable.

(iv) An operator T on Hilbert space H is normal if and only if $\| T^*x\|$ = ... ∀ x ∈ H

(v) If P is perpendicular projection on M Then I – P is a .... on $M^{\perp}$

(vi) if {e i} be complete orthonormal set in Hilbert space and for x ∈ H, x ⊥ {e i} then ....

(vii) If N is normal operator on a Hilbert Space H Then $\|N^2\|$ = ...

(viii) If S_1 and S_2 are non-empty subset of Hilbert space H $S_1 C.S_2$ => ....

(ix) If M is a linear subspace of a normed linear space M and x + M ∈ N/M Then $\| x + M \|$ = ...

(x) Every complete subspace of a normed linear space is ....

2. Answer the following questions- [2x5=10]

(a) Define Banach space with example.

(b) In Hilbert space H prove that - $(\alpha, a\beta - b\Upsilon) = \bar{a}(\alpha, \beta) - \bar{b}(\alpha,\Upsilon)$ ϒ ∀ a, b ∈ c, α, β, λ ∈ H

(c) let $T_1, T_2 \in \beta$ (H) then prove that- $(T_1 + T_2)^* \equiv T_1^* + T_2^*$

(d) Prove that for x, y in Hilbert space H $\| x + y \|^2 = 2 \| x \|^2 + 2 \| y \|^2$

(e) Prove that an orthonormal set can not contain zero verctor.

Section-B

Answer all questions- [12x5=60]

3. State and prove Hahn Banach space.

Or

Show that $l^n{}_P$ is a Banch space.

4. State and prove closed graph Theorem

Or

let N be a non zero normed linear space and S = { x ∈ N : $\| x \| \leq 1$ } be a linear subspace of N. Then prove that N is Banach space if and only if S is complete.

5. let M be a linear subspace of a normed linear space N and let f be a functional defined on M Then f can be extended to a functional f_o on the whole space N such that $\| f \| = \| f_o \|$

Or

let M be a proper closed linear subspace of a Hilbert space H then prove that there exist a non zero vector Zo in H such that Zo ⊥ H

6. state and prove uniform bounded principle.

Or

let ϒ be a fixed vector in a Hilbert space H and let fy be a scalar valued function on H defined by-

fy (x) = (x, y) ∀ x ∈ H

Then prove that fy is a functional I n H* and $\|$ fy $\| = \| y \|$.

7. Prove that A closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

Or

An operator on Hilbert space is normed if and only if it's real and imaginary parts commutes.