PF-361

M.A./M.Sc. Mathematics
3rd Semester Examination, Dec., 2022

Paper - II

Partial Differential Equations, Mechanics and Gravitation-I

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 29

Note : Answer from **both** the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Choose the correct answer of the following : 1×10

(*a*) Which of the following is non-homogeneous heat equation ?

(*i*) $U_t - \Delta U = 0$

(*ii*) $\Delta U = 0$

(*iii*) $U_{tt} - \Delta U = f$

(*iv*) None of these

(*b*) Which of the following is homogeneous wave equation ?

(*i*) $U_t - \Delta U = 0$

(*ii*) $U_{tt} - \Delta U = 0$

(*iii*) $U_{tt} - \Delta U = f$

(*iv*) $\Delta U = f$

(*c*) Which of the following is Poisson equation ?

(*i*) $\Delta U = 0$

(*ii*) $U_t - \Delta U = f$

(*iii*) $\Delta U = f$

(*iv*) $U_{tt} - \Delta U = f$

(*d*) If $\Delta U = 0$, then U is called :

(*i*) Laplace function

(*ii*) Poisson function

(*iii*) Harmonic function

(*iv*) None of the above

(*e*) If $U \in C^2(\Omega) \cap C(\Omega)$ is a harmonic function in a bounded domain Ω, then

$\max_{\Omega} U =$

(*i*) $\min_{\Omega} U$

(ii) $\min_{\Gamma} U$

(iii) $\max_{\Gamma} U$

(iv) None of these

(f) If $\phi(x, t)$ is the fundamental solution of the heat equation, then for $t > 0$; $\int_{R^n} \phi(x,t)\,dx =$

(i) 0

(ii) 1

(iii) 2

(iv) None of these

(g) If $L^{-1}\left\{\dfrac{e^{-5s}}{(s-2)^4}\right\} = F(t)$, then which of the following is ture ?

(i) $F(t)=0;\ t < 5$

(ii) $F(t) = \dfrac{t^3 e^{2t}}{6};\ t > 5$

(iii) $F(t) = 0;\ t > 5$

(iv) None of these

(h) If $\bar{f}_s(P)$ is the finite Fourier sine transform of $f(x)$ over the interval $(0, l)$, then the inversion formula for sine transform is :

(i) $f(x) = \sqrt{\dfrac{2}{\pi}} \int_0^\infty \bar{f}_s(P) \sin px\, dp$

(ii) $f(x) = \sqrt{\dfrac{2}{\pi}} \int_0^\infty \bar{f}_s(P) \cos px\, dp$

(iii) $f(x) = \dfrac{2}{\pi} \sum_{P=1}^{\infty} \bar{f}_s(P) \sin px$

(iv) $f(x) = \dfrac{1}{l} \bar{f}_s(0) + \dfrac{2}{l} \sum_{P=1}^{\infty} \bar{f}_s(P) \sin px$

(i) If the rod of density ρ and cross section k, be of infinite length in both directions then attraction, at any point which is at distance P from the rod is :

(i) $\dfrac{\gamma k \rho}{P}$

(ii) $\dfrac{2\gamma k \rho}{P}$

(*iii*) $\frac{\gamma k\rho}{\rho}$

(*iv*) $\frac{2\gamma k\rho}{P}\sin\left(\frac{APB}{2}\right)$

(*j*) Let M be the mass of spherical shell whose radius is 6m. Then attraction at any point which is at distance 5m from the centre of the shell is :

(*i*) $\frac{\gamma M}{36}$

(*ii*) $\frac{\gamma M}{25}$

(*iii*) 0

(*iv*) None of these

2. Answer the following questions : 2×5

(*a*) Show that the attraction of a solid hemisphere at the centre of its plane base is $\frac{3}{2}\frac{\gamma M}{a^2}$, where M is the mass and 'a' is the radius.

(*b*) Find Laplace transform of $\sin\sqrt{+}$.

(*c*) Find Fourier sine transform of $\frac{e^{-ax}}{x}$.

(*d*) Spheres of constant radius b have their centres on the fixed circle $x^2 + y^2 = a^2, z = 0$. Prove that their envelop is the surface

$$(x^2 + y^2 + z^2 + a^2 - b^2)^2 = 4a^2(x^2 + y^2)$$

(*e*) If $\tilde{f}(P)$ is the complex Fourier transform of $f(x)$, then show that the Fourier transform of $f(x)\cos ax$ is

$$\frac{1}{2}\left[\tilde{f}(P-a)+\tilde{f}(P+a)\right].$$

Section-B

Answer the following long answer type questions : 12×

3. Find the fundamental solution of Laplace equation.

OR

If $U \in C(U)$ satisfies the mean value property for each ball $B(x, r) \subset U$, then show that $U \in C^{\infty}(U)$.

4. State and prove mean value formula for Laplace equation.

OR

State and prove mean value property for heat equation.

5. By using Laplace transform, solve

$$\frac{\partial y}{\partial x} = 2\frac{\partial y}{\partial t} + y,\ y(x,0) = 6e^{-3x}$$

which is bounded for $x > 0$, $t > 0$.

OR

Use finite Fourier transform, solve

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}; U(0,t) = 0;$$

$$U(4,t) = 0, U(x,0) = 2x$$

where $0 < x < 4$, $t > 0$.

6. State and prove D'Alembert's formula.

OR

A vertical solid cylinder of height h, radius a and density ρ bounded by plane ends perpendicular to axis is divided by a plane through the axis into two parts. Show that the horizontal attraction of one part on a particle at the centre of the base is

$$2\gamma h\rho \log\left(\frac{a+\sqrt{a^2+h^2}}{h}\right)$$

7. Show that the attraction of a spherical segment on a unit particle at its vertex is

$$2\pi\gamma\rho h\left[1-\frac{1}{3}\sqrt{\frac{2h}{a}}\right]$$

and that on a unit particle at the centre of the base is

$$\frac{2\pi\gamma\rho h}{3(a-h)^2}\left[3a^2-3ah+h^2-h^{1/2}(2a-h)^{3/2}\right]$$

where a is the radius and h the height of the segment.

OR

State and prove Poisson's equation in Cartesian form.