PE-361

(513) M.A./M.Sc. MATHEMATICS (Third Semester)
EXAMINATION, DEC.-2021

Paper - II

PARTIAL DIFFERENTIAL EQUATIONS, MECHANICS AND GRAVITATION–I

Time : Three hours] **[Maximum Marks : 80**

Note : *Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.*

Section–A

1. Answer the following very short answer type questions : **1×10=10**

(1) Laplace equation is called..... .

(2) Heat equation, also known as the..... .

(3) Write formula for one-dimensional wave equation.

(4) Find the degree of P.D.E. :

$$\frac{\partial^2 z}{\partial t^2} = C^2 \frac{\partial^2 z}{\partial y^2}$$

(5) Find $L\{F(t)\}$ where $F(t) = 1$

(6) Find $L\{F(t)\}$ where $F(t) = \text{coshat}$

(7) If P be a particle of unit mass and Q another particle of mass and r the distance between then force of attraction is.....

(8) Find order and degree of P.D.E.

$$\left(\frac{\partial z}{\partial x}\right)^3 + \frac{\partial z}{\partial x} = 0$$

(9) Find the order of P.D.E.

$$\frac{\partial^2 z}{\partial x^2} + xy\frac{\partial z}{\partial x} + y = 0$$

(10) Find the Laplace transform of the function :

$$F(t) = t^n, n = 0, 1, 2, 3, ...$$

2. Answer the following short answer type questions : **2×5=10**

(1) Write short notes on Legendre's transformation. (only formula)

(2) Find the complete integral $(p^2 + q^2)x = pz$.

(3) Find the Laplace transform of the function $F(t) = \sin t \cos t$ for if $p \neq n$.

(4) Find the Fourier cosine transform of $f(x)$ if $f(x) = \sin nx$.

(5) Find the attraction of a uniform solid right circular cone of height h and verticle angle 2α at the vertex of the cone.

Section–B

Answer long type questions : **12×5=60**

3. Find the Fundamental solution of Heat equations.

OR

[P.T.O.]

To find the attraction of a thin uniform rod at an external point.

4. Find the complete integral :

$$2z - px^2 - 2qxy + pq = 0$$

OR

Solve :

$$p = (z + 9y)^2$$

5. Find the solution of wave equation by spherical means for $n = 1$.

OR

A frustum of a uniform thin hollow cone attracts a particle placed at the vertex, show that attraction is $2\pi Ye \sin\alpha \cos\alpha \log \frac{R}{r}$.

where R and r are the radii of the circular ends, α the semi-verticle angle and e the surface density.

6. Solve : $\frac{\partial y}{\partial t} = 2\frac{\partial^2 y}{\partial x^2}$

where $y(0, t) = 0 = y(5, t)$ and
$y(x, 0) = 10 \sin 4\pi x$.

OR

Find :

$$L^{-1}\left\{\frac{1}{(p+1)(p^2+1)}\right\}$$

7. Show that the attraction of a solid hemisphere at the centre of its plane base is $\frac{3}{2} Y \frac{M}{a^2}$ where M is the mass and a is the radius.

OR

Evaluate :

$$L^{-1}\left\{\frac{p+1}{p^2+6p+25}\right\}$$