PF-364

M.A./M.Sc. Mathematics

3rd Semester Examination, Dec., 2022

Paper - V

Operation Research - I

Time : Three Hours] [*Maximum Marks* : **80**

Note : Answer from **both** the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Answer the following questions : 1×10

(*a*) The statement "O.R. is a scientific knowledge through interdisciplinary team effort for the purpose of determining the best utilization of limited resources," is given by ..... .

(*b*) Draw graph of the constraint $x_1 + x_2 \leq 6$, $x_1 \geq 0$, $x_2 \geq 0$.

(*c*) "For every primal constraint, there is a dual variable." This statement is correct or not ?

(*d*) if x_0 is an optimal solution to the primal and B is the primal optimal basis, then an optimal solution to the dual is given by ..... .

(*e*) Define parametric linear programming problem.

(*f*) What is Optimality test ?

(*g*) When does a transportation problem have a unique solution ?

(*h*) What is a balanced transportation problem ?

(*i*) What is Network Scheduling ?

(*j*) What is full form of CPM ?

2. Answer the following questions : 2×5

(*a*) What is Judgement phase of scientific method in O.R. ?

(*b*) Write the dual of the L.P.P. :

Minimize $z = 4x_1 + 6x_2 + 18x_3$

Subject to the constraints :

$$x_1 + 3x_2 \geq 3$$

$$x_2 + 2x_3 \geq 5$$

and $x_1, x_2, x_3 \geq 0$

(*c*) What is goal programming ?

(*d*) Write steps of Vogel's approximation method.

(*e*) Write any two distinction between PERT and CPM.

Section-B

Answer the following questions : 12×5

3. Use the graphical method to solve the following L.P.P. :

Minimize $z = -x_1 + 2x_2$

Subject to the constraints :

$$-x_1 + 3x_2 \leq 10$$

$$x_1 + x_2 \leq 6$$

$$x_1 - x_2 \leq 2$$

and $x_1 \geq 0, x_2 \geq 0$

OR

State and prove the fundamental theorem of linear programming.

4. Use duality to solve the following L.P.P. :

Maximize $z = 2x_1 + x_2$

Subject to the constraints :

$$x_1 + 2x_2 \leq 10$$

$$x_1 + x_2 \leq 6$$

$$x_1 - x_2 \leq 2$$

$$x_1 - 2x_2 \leq 1$$

and $x_1, x_2 \geq 0$

OR

Use dual simplex method to solve the following L.P.P. :

Minimize $z = 3x_1 + x_2$

Subject to the constraints :

$$x_1 + x_2 \geq 1$$

$$2x_1 + 3x_2 \geq 2$$

and $x_1, x_2 \geq 0$

5. Consider the parametric L.P.P. :

Maximize $z = 3x_1 + 2x_2 + 5x_3$

Subject to the constraints :

$$x_1 + 2x_2 + x_3 \leq 430 + \mu$$

$$3x_1 + 2x_3 \leq 460 - 4\mu$$

$$x_1 + 4x_2 \leq 420 - 4\mu$$

and $x_1, x_2, x_3 \geq 0$

Determine the critical values (range) of μ for which the solution remains optimum basic feasible.

OR

Solve the following linear goal programming problem graphically :

Find x_1 and x_2 so as to :

Minimize $z = G_1\left(d_3^+ + d_4^+\right) + G_2 d_1^+ + G_3 d_2^-$

$$+G_4\left(d_3^- + \frac{3}{2}d_4^-\right) \text{ and}$$

Satisfy the goals :

$$G_1 : x_1 + x_2 + d_1^- + d_1^+ = 40$$

$$G_2 : x_1 + x_2 + d_2^- - d_2^+ = 100$$

$$G_3 : x_1 + d_3^- - d_3^+ = 30$$

$$G_4 : x_2 + d_4^- - d_4^+ = 15$$

$x_i, d_i^-, d_i^+ \geq 0$, for all $i = 1, 2, 3, 4$

The goals have been listed in order of priority.

6. Obtain an initial basic feasible solution to the following T.P. using the Vogel's approximation method :

Warehouse	Stores				Availability
	I	II	III	IV	
A	5	1	3	3	34
B	3	3	5	4	15
C	6	4	4	3	12
D	4	–1	4	2	19
Requirement	21	25	17	17	80

OR

How the problem of degeneracy arises in a transportation problem ? Explain how does one overcome it ?

7. "A transportation problem is a minimum cost flow problem." Explain.

OR

A project consists of a series of tasks labelled *A*, *B*, ..., *H*, *I* with the following relationships ($W < X$, Y means X and Y cannot start until W is completed; $X, Y < W$ means W cannot start until both X and Y are completed). With this notation construct the network diagram having the following constraints :

$A < D, E$; $B, D < F$; $C < G$; $B, G < H$; $F, G < I$

Find also the minimum time of completion of the project, when the time (in days) of completion of each task is as follows :

Task	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
Time	23	8	20	16	24	18	19	4	10