PF-363

M.A./M.Sc. Mathematics
3rd Semester Examination, Dec., 2022

Paper - IV

Fuzzy Sets and Their Applications-I

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer from **both** the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Answer the following questions : 1×10

(*a*) Given two fuzzy sets, A and B, their standard intersection, $A \cap B$ and standard union $A \cup B$ are defined for all $x \in X$ by the equations ....

(*b*) Given a fuzzy set A defined on X and any $\alpha \in [0, 1]$, ${}^{\alpha+}A = ?$

(*c*) Give example of a fuzzy complement that is continuous but not involutive.

(*d*) For each $a,\ b \in [0, 1]$, drastic union $u(a, b) = ?$

(*e*) If A and B are fuzzy numbers, then for all $z \in R$, $(A \cdot B)(z) = ?$

(*f*) If R is the set of all fuzzy numbers, then define absorption laws for MIN and MAX operations from $R \times R \to R$.

(*g*) Given a fuzzy relation $R(X, Y)$, its domain is defined by ....

(*h*) Every fuzzy relation R can be uniquely represented in terms of its α-cuts by the formula ....

(*i*) Write any one equation from the matrix equation

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \end{bmatrix} \circ \begin{bmatrix} .9 & .5 \\ .7 & .8 \\ .1 & .4 \end{bmatrix} = \begin{bmatrix} .6 & .3 \\ .2 & .1 \end{bmatrix}$$

(j) Let the t-norm i employed in the fuzzy relation equation $P \overset{i}{\circ} Q = R$ be the product and let $Q = \begin{bmatrix} .1 \\ .2 \\ .3 \end{bmatrix}$ and $R = \begin{bmatrix} .12 \\ .18 \\ .27 \end{bmatrix}$. Then $\hat{P} = ?$

2. Answer the following questions : 2×5

(a) State first decomposition theorem.

(b) Define t-conorm.

(c) If A and B are two fuzzy numbers on X defined by

$$A(x) = \begin{cases} 0, & \text{if } x \le 3 \text{ and } x > 5 \\ x-3, & \text{if } 3 < x \le 4 \\ 5-x, & \text{if } 4 < x \le 5 \end{cases}$$ and

$$B(x) = \begin{cases} 0, & \text{if } x \le 12 \text{ and } x > 32 \\ (x-12)/8, & \text{if } 12 < x \le 20 \\ (32-x)/12, & \text{if } 20 < x \le 32 \end{cases}$$

then find the solution of the fuzzy equation $A \cdot X = B$.

(d) Explain strong fuzzy homomorphism.

(e) If $S(Q, R) \neq \phi$, then prove that $\hat{P} = R \overset{w_i}{\circ} Q^{-1}$ is the greatest member of $S(Q, R)$.

Section-B

Answer any **five** of the following questions : 12×5

3. Complete the saclar cardinalities for each of the following fuzzy sets :

(a) $C(x) = \frac{x}{x+1}, x \in \{0, 1, ..., 10\}$

(b) $D(x) = 1 - \frac{x}{10}, x \in \{0, 1, ..., 10\}$

and then prove that $|C| + |D| = |C \cup D| + |C \cap D|$, where $\cap$, $\cup$ are the standard fuzzy intersection and union respectively.

4. Let A, $B \in \Im(X)$ and $\alpha \in [0, 1]$. Then prove the following :

(a) ${}^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B$

(b) ${}^{\alpha}(\overline{A}) = {}^{(1-\alpha)+}\overline{A}$

5. For all a, $b \in [0, 1]$ prove that

$$i_{\min}(a,b) \le i(a,b) \le \min(a,b)$$

where $i_{\min}$ denotes the drastic intersection.

6. State and prove fundamental theorem of t-conorms.

7. Explain the four basic arithmetic operations on fuzzy numbers with example.

8. Let R denote the set of all fuzzy numbers. Let MIN and MAX are operations from $R \times R \to R$ defined by

$$\text{MIN}(A,B)(z) = \sup_{z=\min(x,y)} \min[A(x), B(y)]$$

$$\text{MAX}(A,B)(z) = \sup_{z=\max(x,y)} \min[A(x), B(y)],$$

$$z \in R$$

Then prove that <R, MIN, MAX> is a distributive lattice.

9. Define transitive fuzzy closure. Also determine the transitive max-min closure $R_T(X, X)$ for a fuzzy relation $R(X, X)$ defined by the membership matrix

$$R = \begin{bmatrix} .7 & .5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & .4 & 0 & 0 \\ 0 & 0 & .8 & 0 \end{bmatrix}$$

10. Determine partition trees for the following similarity relation $R(X, X)$:

	a	b	c	d	e	f	g
a	1	.8	0	.4	0	0	0
b	.8	1	0	.4	0	0	0
c	0	0	1	0	1	.9	.5
d	.4	.4	0	1	0	0	0
e	0	0	1	0	1	.9	.5
f	0	0	.9	0	.9	1	.5
g	0	0	.5	0	.5	.5	1

11. Explain problem partitioning with example.

12. Solve the following fuzzy relation equation for max-min composition :

$$P \circ = \begin{bmatrix} .9 & .6 & 1 \\ .8 & .8 & .5 \\ .6 & .4 & .6 \end{bmatrix} = \begin{bmatrix} .6 & .6 & .5 \end{bmatrix}.$$
