PF-365

M.A./M.Sc. Mathematics

3rd Semester Examination, Dec., 2022

Paper - IV

Fluid Mechanics-I

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer from **both** the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Answer the following questions : 1×10

(*a*) Write the differential equation for path lines.

(*b*) If the motion is irrotational, then curl $\bar{q} = ?$

(*c*) Define streamlines.

(*d*) Write the energy equation for incompressible fluid.

(*e*) Velocity potential ϕ satisfies which equation?

(*f*) Write the main difference between Eulerian and Lagrangian method.

(*g*) Write the Euler's equation of motion in *x*-direction.

(*h*) Write the statement of Milne-Thomson circle theorem.

(*i*) Define source and sink.

(*j*) Explain image system.

2. Answer the following questions : 2×5

(*a*) Show that $\frac{x^2}{a^2} f(t) + \frac{y^2}{b^2} f(t) = 1$ is possible form of boundary surface.

(*b*) Find the equation of continuity for cylindrical symmetry.

(*c*) Show that the curve of $\phi(x, y)$ = constant and $\psi(x, y)$ = constant cut orthogonally at their point of intersection.

(*d*) Discuss the physical significance of streamlines.

(*e*) What arrangement of source and sink will give rise to the function $w = \log\left(z - \frac{a^2}{z}\right)$?

Section-B

Answer the following questions : 12×5

3. Find the equation of continuity in spherical polar coordinates.

OR

Find the Euler's equation of motion in vector form.

4. If every particle moves on the surface of a sphere, prove that the equation of continuity is $\frac{\partial \rho}{\partial t}\cos\theta+\frac{\partial}{\partial\theta}(\rho w\cos\theta)+\frac{\partial}{\partial\phi}(\rho w'\cos\theta)=0$ ρ being the density, θ and ϕ the latitude and longitude of any element and w and w' the angular velocity of the element in latitude and longitude respectively.

OR

An infinite fluid in which a spherical hollow shell of radius a is initially at rest under the action of no forces. If a constant pressure π is applied at infinity, show that the time of filling up the cavity is $\pi^2 a\left(\frac{\rho}{\pi}\right)^{\frac{1}{2}} 2^{\frac{5}{6}}\left\{\Gamma\left(\frac{1}{3}\right)\right\}^{-3}$.

5. State and prove Bernoulli's theorem due to streamlines.

OR

In the case of the two dimensional fluid motion produced by a source of strength m placed at a point S outside a rigid circular disc of radius a whose centre is O, show that the velocity of slip of the fluid in contact with disc is greatest at the points where the

lines joining S to the ends of diameter at right angles to OS. Cut the circle and prove that its magnitude of these points is $\frac{2m\cdot OS}{\left(OS^2-a^2\right)}$.

6. Find the image of a doublet in circle.

OR

Between the fixed boundaries $\theta=\frac{\pi}{6}$ and $\theta=\frac{-\pi}{6}$ there is a two dimensional liquid motion due to a source at the point $(r=c,\ \theta=\alpha)$ and a sink at the origin, absorbing water at the same rate as the source produce it. Find the stream function and show that one of the streamline is a part of the curve. $r^3\sin 3\alpha=c^3\sin 3\theta$.

7. State and prove Blasius theorem.

OR

Find the velocity potential and stream function at any point of a liquid contained between two coaxial cylinder of radii a and b $(a<b)$ when the cylinders are moved suddenly parallel to themselves in direction as right angles with velocities U and V respectively.