PE-360

M.A./M.Sc. MATHEMATICS (Third Semester)

EXAMINATION, DEC.-2021

Paper - I

INTEGRATION THEORY AND FUNCTIONAL ANALYSIS (I)

Time : Three hours] **[Maximum Marks : 80**

Note : *Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.*

Section–A

1. Answer the following questions : **1×10=10**

(1) Let $A \subseteq E \subseteq X$ such that A is measurable and $u(A) \leq 0$, then A ....

(2) A countable union of positive set is......... .

(3) Every signed measure is.........of two measures.

(4) The relation of absolutely continuous is......... and ....

(5) Let $f : X \to [-\infty, \infty]$ then the Radon Nikodym derivative of the measure $v, w.r$ to μ is denoted by......... .

(6) The σ algebra A^* is ....

(7) If $A \subseteq X$ and $B \subseteq Y$ then $A \times B \subseteq X \times Y$ is called......... .

(8) If (X, A_1) and (Y, A_2) are measurable spaces, then.........is again a measurable space.

(9) Every section of measurable function is......... .

(10) Every absolutely continuous function f defined on $[a, b]$ is......... .

2. Answer the following questions : **2×5=10**

(1) Define signed measure.

(2) If v is a finite signed measure and if μ is a signed measure such that $v << \mu$ corresponding to every positive number ε there is a positive number δ such that $|v|(E) < \varepsilon$ for every measurable set E with $|\mu|(E) < \delta$.

(3) Write the statement of Tonelli's theorem.

(4) Let $\{Ai \times Bi\}$ be a countable disjoint collection of measurable rectangles whose union is measurable rectangle $A \times B$. Then

$$\lambda(A \times B) = \Sigma\lambda(Ai \times Bi)$$

(5) Let μ be a Baire measure on a locally compact space X and E is a σ bounded Baire set in X. Then for $\varepsilon > 0$, there is a σ compact open set O with $E \subseteq O$ and $\mu(0 \sim E) < \varepsilon$.

Section–B

Answer all questions : **12×5=60**

3. State and prove Hahn decomposition theorem.

OR

If μ is finite Baire measure on the real line, then its cumulative distribution function F is monotone increasing bounded function which is continuous on right and $x \xrightarrow{\lim} -\infty F(x) = 0$.

[P.T.O.]

4. State and prove Lebesgue decomposition theorem.

OR

Let μ be a measure defined on σ algebra. A containing the Baire sets. Assume that either μ is quasi regular or μ is inner regular. Then for each $E \in A$ with $\mu(E) < \infty$ there is a Baire set B with $\mu(E \Delta B) = 0$.

5. The intersection of a sequence of inner regular sets of finite measure is inner regular. Also, the intersection of a decreasing sequence of outer regular sets of finite measure is outer regular.

OR

State and prove caretheodory extension theorem.

6. Using Fubini's theorem, show that :

$$\int_0^1 dx \int_0^1 \frac{x^2 - y^2}{x^2 + y^2}\, dy = \int_0^1 dy \int_0^1 \frac{x^2 - y^2}{x^2 + y^2}\, dx$$

OR

Let μ be a borel measure which is finite on compact sets. Then the following are equivalent :

(1) μ is outer regular on σ bounded sets

(2) μ is inner regular on σ bounded sets

7. State and prove Tunnell's theorem.

OR

State and prove Riesz representation theorem.