PD-360-S.E.-CV-19

M.A./M.Sc. MATHEMATICS (3rd Semester)

Examination, Dec.-2020

Paper-I

INTEGRAION THEORY AND FUNCTIONAL ANALYSISI-I

Time : Three Hours] **[Maximum Marks : 80**

[Minimum Pass Marks : 29

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. **Answer the following questions:-** 1x10=10
   (a) If μ is signed measure on a measurable space (X, A) then $\mu(\phi) =$ ....
   (b) A signed measure μ is said to be totally finite if............
   (c) $\mu << V$ and $v(A)=0, \forall A \in A$ Then $\mu(A)=$ ....
   (d) Any bounded linear functional on Lp can be written as the difference of two......
   (e) If μ and v are any two measures on a σ ring then v is absolutely continuous with respect to...........
   (f) A set E is said to be of the type $F\sigma$ if..........
   (g) Every Borel set is σ-bounded if and only if..........
   (h) A set E in $\hat{s}$ is said to be inner regular with respect to μ if..........
   (i) Define section of a set.
   (j) Define content.
2. **Answer the following questions:-** 2x5=10
   (a) Prove that every section of a measurable set is measurable set.
   (b) Let E and F are measurable set and μ is a signed measure such that $E \subset F$ and $|\mu(F)| < \infty$ then prove that $|\mu(E)| < \infty$
   (c) If $E_1 = A_1 \times B_1$ and $E_2 = A_2 \times B_2$ are non empty rectangle then $E_1 \subset E_2$ if and only if $A_1 \subset A_2$ and $B_1 \subset B_2$
   (d) Prove that a Borel measurable set is lebesgue measurable.
   (e) If μ is the Borel measure induced by a regular content λ, then prove that $\mu(c) = \lambda(c)$ for every c

Section-B 12x5=60

Answer all questions.

3. State and prove Riesz representation theorem.

   OR

   If E is a measurable set with finite negative measure. Then prove that E contains a negative set A with $\mu(A) < 0$
4. State and prove Radon-Nikodym theorm.

   OR

   If $< E_n >$ is a disjoit sequence in A^* then prove that
   $$\mu^*(\cup_{n=1}^{\infty} E_n) = \sum_{n=1}^{\infty} \mu^*(E_n)$$
5. Using Fubini's theorem verify.
   $$\int_0^1 \left\{\int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2} dx\right\} dy \neq \int_0^1 \left\{\int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2} dy\right\} dx$$

   OR

   Let $\{A_K \times B_K\}_{K=1}^{\infty}$ be a disjoint collection of measurable rectangles whose union is a measurable rectangle $A \times B$ then prove that $\mu(A) \times \mu(B) = \sum_{K=1}^{\infty} \mu(A_K) \times \mu(B_K)$
6. Prove that a function f is of bounded variation if and only if it can be expressed as a difference of two monotonic function both non-decreasing.

   OR

   Prove that every compact Bair set is G_s type.

7. If μ_0 is a Baire measure and if for every c in
$\lambda(c) = inf\{\mu_0(u_0: c \subseteq u_0)\}$ then prove that λ is a regular content.

OR

Prove that inner contents $\lambda *$ induced by a content λ vanishes at 0 and is monotonic, countably sub additive and countably additive.