PE-363

(513) M.A./M.Sc. MATHEMATICS (Third Semester) EXAMINATION, DEC.-2021

FUZZY SETS AND THEIR APPLICATIONS-I

Time : Three hours] **Paper - IV** **[Maximum Marks : 80**

Note : *Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.*

Section–A

1. Answer the following questions : **1×10=10**

(1) If A is a fuzzy set given by

$$A(x) = \begin{cases} 0 & \text{when} \quad x \le 20 \text{ or } \ge 60 \\ \dfrac{(x-20)}{15} & \text{when} \quad 20 < x < 35 \\ \dfrac{(60-x)}{15} & \text{when} \quad 45 < x < 60 \\ 1 & \text{when} \quad 35 \le x \le 45 \end{cases}$$

then ${}^{0.2}A = ?$

(2) If A is a fuzzy set defined by :

$$A = \frac{\cdot 2}{x_1} + \frac{\cdot 8}{x_2} + \frac{\cdot 6}{x_3} + \frac{\cdot 3}{x_4}$$

then find $h(A)$. Is this a normal fuzzy set ?

(3) If A and B are fuzzy sets on $X = \{1, 2, 3, 4, 5\}$ given by

$A = \frac{\cdot 2}{1} + \frac{\cdot 3}{2} + \frac{\cdot 8}{3} + \frac{\cdot 4}{4} + \frac{\cdot 8}{5}$ and $B = \frac{\cdot 3}{1} + \frac{\cdot 1}{2} + \frac{\cdot 9}{3} + \frac{\cdot 3}{4} + \frac{\cdot 7}{5}$

then find standard union of A and B.

(4) If A and B are fuzzy sets on $X\{x_1, x_2, x_3, x_4\}$ given by $A = \frac{\cdot 2}{x_1} + \frac{\cdot 8}{x_2} + \frac{\cdot 4}{x_3} + \frac{\cdot 7}{x_4}$ and

$B = \frac{\cdot 3}{x_1} + \frac{\cdot 6}{x_2} + \frac{\cdot 7}{x_3} + \frac{\cdot 9}{x_4}$ then find ${}^{\cdot 6}(A \cap B)$.

(5) If $i : [0,1] \times [0,1] \to [0,1]$ is a t-norm, then bounded difference $i\,(0.4, 0.3) = ?$

(6) Let an involutive function $C : [0,1] \to [0,1]$ satisfy monotonicity, then prove that C satisfies boundary condition.

(7) Define dual point of $a \in [0, 1]$ with respect to a fuzzy complement C.

(8) Evaluate :

$$[4,6] \div [1,2] \quad \text{and} \quad [3, 5] - [4, 5]$$

(9) Draw saggital diagram for the fuzzy relation given below

$$X \quad \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \!\!\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} \cdot 7 & 0 & 0\cdot 3 & 0 \\ 0 & \cdot 7 & 1 & 0 \\ \cdot 9 & 0 & 0 & 1 \\ \cdot 3 & 0 & \cdot 6 & \cdot 4 \end{bmatrix} \end{array}$$

(10) Define an i-transitive fuzzy relation.

[P.T.O.]

2. Answer the following questions : **2×5=10**

(1) Prove that law of absorption holds in fuzzy sets.

(2) Let $A, B \in \exists(x)$, then prove that $A \subseteq B \Rightarrow {}^{\alpha}A \subseteq {}^{\alpha}B$.

(3) If A is a fuzzy set given by $A = \frac{\cdot 2}{x_1} + \frac{\cdot 4}{x_2} + \frac{\cdot 6}{x_3} + \frac{\cdot 8}{x_4} + \frac{1}{x_5}$ find $\cdot 6^{A}$.

(4) If A is a fuzzy number given by

$$A = \begin{cases} 0 & \text{for } \quad x \le 1 \,\&\, x > 5 \\ (x+1)/2 & \text{for } \quad 1 < x \le 3 \\ (5-x)/2 & \text{for } \quad 3 < x \le 5 \end{cases}$$ then find $\cdot 2A$.

(5) For any $a, b, d \in [0,1]$, prove that $i(a,b) \le d$ iff $w_i(a,d) \ge d$

Section–B

Answer any five of the following questions : **12×5=60**

3. Let $f : X \to Y$ be an arbitrary crisp function. Then for any $A \in f(X)$ and all $\in [0,1]$ the property ${}^{\alpha}[f(A)] \supseteq f({}^{\alpha}A)$ of f fuzzified by the extension principle holds, but not conversely.

4. State and prove first characterization theorem of fuzzy complements.

5. Prove that $(ab, a+b-ab, C_S)$ where C_S is the standard complement, is a dual triple but $(ab, \max(a,b), C_S)$ is not a dual triple.

6. Consider fuzzy sets A and B whose membership functions are defined by formulae $A(x) = \frac{x}{(x+1)}$, for all $x \in \{0,1,2,\ldots,10\} = X$. Calculate scalar cardinality and fuzzy cardinality of A.

7. Let A and B be fuzzy numbers defined by

$$A = \frac{\cdot 2}{[0,1)} + \frac{\cdot 6}{[1,2)} + \frac{\cdot 8}{[2,3)} + \frac{\cdot 9}{[3,4)} + \frac{1}{4} + \frac{\cdot 5}{(4,5]} + \frac{\cdot 1}{(5,6]} \text{ and}$$

$$B = \frac{\cdot 1}{[0,1)} + \frac{\cdot 2}{[1,2)} + \frac{\cdot 6}{[2,3)} + \frac{\cdot 7}{[3,4)} + \frac{\cdot 8}{(4,5]} + \frac{\cdot 9}{(5,6]} + \frac{1}{6} + \frac{\cdot 5}{(6,7]} + \frac{\cdot 4}{(7,8]} + \frac{\cdot 2}{(8,9]} + \frac{\cdot 1}{(9,10]}$$

then solve the fuzzy equation $A + X = B$.

8. Find the transitive max-min closure $R_T(X,X)$ for a fuzzy relation $R(X,X)$ defined by the membership matrix

$$R = \begin{bmatrix} \cdot 7 & \cdot 5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \cdot 4 & 0 & 0 \\ 0 & 0 & \cdot 8 & 0 \end{bmatrix}$$

9. Let R be a reflexive fuzzy relation on X^2, where $|X| = n \ge 2$, then prove that

$$R_{T(i)} = R^{(n-1)}$$

10. Solve the following fuzzy relation equations for the max-min. composition :

$$p \circ \begin{bmatrix} \cdot 5 & \cdot 7 & 0 & \cdot 2 \\ \cdot 4 & \cdot 6 & 1 & 0 \\ \cdot 2 & \cdot 4 & \cdot 5 & \cdot 6 \\ 0 & \cdot 2 & 0 & \cdot 8 \end{bmatrix} = [\cdot 5 \;\; \cdot 5 \;\; \cdot 4 \;\; \cdot 2]$$