PE-365

(513) M.A./M.Sc. MATHEMATICS (Third Semester)
EXAMINATION, DEC.-2021
FLUID MECHANICS-I (Paper - IV)

Time : Three hours] **[Maximum Marks : 80**

Note : *Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.*

Section–A

1. Answer the following questions : **1×10=10**

(1) The equation of stream lines is.....

(2) Define conservative field of force.

(3) For incompressible flow we have

(a) div $q = 0$ (b) $\frac{dq}{dt} = 0$ (c) $\frac{Dq}{dt} = 0$ (d) None of these

(4) The relation between ϕ and ψ is :

(a) $\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$ and $\frac{\partial\phi}{\partial y} = \frac{\partial\psi}{\partial x}$ (b) $\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$ and $\frac{\partial\phi}{\partial y} = \frac{-\partial\psi}{\partial x}$

(c) $\frac{\partial\phi}{\partial x} = \frac{-\partial\psi}{\partial y}$ and $\frac{\partial\phi}{\partial y} = \frac{\partial\psi}{\partial x}$ (d) None of these

(5) Define two dimensional sources.

(6) For circulation about a circular cylinder the complex potential w is given by :

(a) $\frac{ik}{2\pi}\log z$ (b) $\frac{2\pi}{ik}\log z$ (c) $\frac{2k}{i\pi}\log z$ (d) $\frac{2i}{\pi k}\log z$

(7) Define Steady motion.

(8) Define Rotational motion.

(9) The condition that the surface $f(x, y, zt) = 0$ may be a boundary surface is.....

(10) Define Normal component of velocity for the boundary.

2. Answer the following questions : **2×5=10**

(1) Define velocity potential.

(2) Find the Boundary surface condition.

(3) Define stream function or current function in two dimensions.

(4) To find the complex potential for a two dimensional source of strength m placed at the origin.

(5) To discuss the general motion of a cylinder of any cross section. (Only formula for ψ function)

Section–B

Answer the following questions : **12×5=60**

3. If the velocity of an incompressible fluid is given by $u = \frac{3x^2 - r^2}{r^5}$, $v = \frac{3xy}{r^5}$, $w = \frac{3xz}{r^5}$ then prove that liquid of motion is possible.

OR

[P.T.O.]

Show that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t - 1 = 0$ is a possible form of boundary surface and find an expression for normal velocity.

4. If ω is the area of cross-section of a stream filament, prove that the equation of continuity is

$$\frac{\partial}{\partial t}(\rho\omega) + \frac{\partial}{\partial S}(\rho\omega q) = 0$$

OR

A mass of fluid is in motion so that the lines of motion lie on the surface of coaxial cylinder; show that the equation of continuity is $\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{(\partial \rho V_\theta)}{\partial \theta} + \frac{\partial(\rho V_z)}{\partial Z} = 0$

where V_θ, V_z are the velocities perpendicular and parallel to Z.

5. Define Euler's equation of motion in Vector Form.

OR

Air obeying Boyle's law, is in motion in a uniform tube of small section.

Prove that if ρ be the density and V the velocity at a distance x from a fixed point at time t $\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}[V^2 + K)\rho]$ where $K = \frac{p}{\rho}$

6. A sphere of radius a is surrounded by infinite liquid of density ρ, the pressure at infinity being Π. The sphere is suddenly annihilated. Show that pressure at a distance r from the centre immediately falls to $\Pi\left(1 - \frac{a}{r}\right)$.

OR

To find the image of a simple source w.r.t. a plane (straight line) and show that the image of a doublet w.r.t a plane is an equal doublet symmetrically placed.

7. What arrangement of sources and sinks will give rise to the function $W = \log\left(z - \frac{a^2}{z}\right)$?

Draw a rough sketch of the stream lines in this case and prove that two of them sub-divide into the circle $r = a$ and axis of y.

OR

To discuss the motion of a circular cylinder moving with velocity U along x-axis in an infinite mass of liquid at rest at infinity.