AI-1546

M.A./ M.Sc. (Final) Mathematics

Term End Examination, 2020-21

Compulsory/Optional

Group-

Paper-

PARTIAL DIFFRENTIAL EQUATIONS, MECHANICS & GRAVITATION

Time:- Three Hours] [Maximum Marks:100

[Minimum Passing Marks: 036

Note: Answer any five questions. All questions carry equal marks.

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1. (a) Solve $(D^2 + 6D + 5)y = e^{-t}$by Using Laplace transform. Where$y(0) = 0, y'(0) = 1$.

   (b) If $u \in C^2(U)$ is harmonic then

$$u(x) = \int_{\partial B(x,r)} uds = \int_{B(x,r)} udy$$

   for each ball $B(x,r) \subset U$

2. (a) State and prove symmetry of Green's function.

   (b) Solve the partial differential equation $x^2p + y^2q = z^2$

3. (a) State and prove fundamental solution of Heat equation.

   (b) Derive the Kirchhoff's formula for wave equation.

4. (a) State and prove Hopt Lax formula.

   (b) State and prove Donkin's theorem.

5. (a) Derive Langrange's equations of second kind.

   (b) State and prove fundamental lemma of calculus of variations.

6. (a) Define Poisson bracket and show that

$$[u,[v,w]] + [v,[w,u]] + [w,[u,v]]$$

   (b) State and prove Caucky-kavalevskaya theorem for power series.

7. (a) Find the attraction of a thin uniform rod AB on an external point P.

   (b) Potential of a uniform circular plate at its centre proportional to its radius.

8. (a) Derive Hamiltonian as the total energy of the system.

   (b) Derive Routh's equation of motion.

9. (a) Prove that $\Delta w = 0$ where$w = \int_{t_1}^{t_2} 2Tdt$, T is kinetic energy.

   (b) Prove that the necessary and sufficient condition that the linear transformation

$$Q_i = Q_i(q_i, p_i, t)\ ;\ p_i = p_i(q_i, p_i, t)$$

   May represent canonical transformation is that

$$\sum_{t=1}^{n} p_i q_i - H = \sum_{t=1}^{n} p_i Q_i - K + \frac{df}{dt}$$

   Where F is an arbitrary function of old and new co-ordinations and time t.

10. (a) Prove that the attraction of a uniform thin rectangular phase of mass M upon an Unit mass at P situated on a perpendicular to the plate through its centre is

$$\frac{MY}{ab}\sin^{-1}\frac{ab}{\sqrt{(h^2 + a^2)(h^2 + b^2)}}$$

   (b) Define equipotential surface and show that the attraction at any point P is Normal to the equipotential surface which pass is through P.