AI-1550
M.A./M.Sc. (Final) Mathematics
Examination- MAR-APR 2021
Compulsory/Optional
Paper-VI
Paper Title: **Fluid Mechanics**

Time:- Three Hours] **[Maximum Marks:100**
[Minimum Marks: 036

Note: Answer any five questions. All question carry equal marks.

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1. Determine the acceleration at the point (2,1,3) at t=0.5sec.If $u = yz + t, v = xz + t$ and $w = xy$.
2. Amass of fluid is in motion so that the liner of motion lie on the surface of coaxial cylinder, show that the equation of continuity is $\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho U_\theta)}{\partial \theta} + \frac{\partial(\rho V_z)}{\partial z} = 0$where$V_\theta$,$V_z$, are the velocities perpendicular and parallel to Z.
3. If the velocity of an incompressible fluid at the point (x,y,z) is given by $\left(\frac{3xy}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right)$prove that the liquid motion is possible and the velocity potential is $\frac{\cos\theta}{r^2}$.
4. Show that the ellipsoid$\frac{x^2}{a^2k^2t^{2n}}$+$kt^n\left(\frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 1$is a possible from of the boundary surface of a liquid.
5. Show that $\int \frac{dp}{e} + \frac{1}{2}q^2 + V = C$ where the motion is steady and the velocity potential does not exist, V being the potential from which the external forces are derivable.
6. Air obeying Boyle's law is in motion in a uniform tube of small section, prove that if e be the density and v the velocity at a distancex from a fixed point at time t $\frac{\partial^2 e}{\partial t^2} - \frac{\partial^2}{\partial x^2}\{(V^2 + k)e\}$
7. Find complex Potential for a Source.
8. Use the method of images to prove that if there be a surface m at point in a fluid bounded by the liner $\theta = 0$ and $\theta = \frac{1}{3}\pi$ the solution is $\emptyset + i\psi = -m\log\{(z^3 - {z_0}^3)\left(z^3 - {z'_0}^3\right)\}$where $z_0 = x_0 + iy_0$and $z'_0 = x_0 + iy_0$.
9. A sphere of radius a is moving with constant velocity U through an infinite liquid at rest at infinity. If Po be the pressure at infinity show that the pressure at any point of the surface of the sphere, the radius to which point makes an angleθ with the direction of motion is given by$P = Po + \frac{1}{2}eu^2\left(I + \frac{9}{4}\right)sin^2\theta$.
10. Show that the flux of velocity through any cross-section of a vertex tube is a constant all along the tube.