AH 1549 CV-19

M.A./M.Sc. (Final)

Term End Examination, 2019-20

MATHEMATICS

Paper- V

Difference Equations

Time:- Three Hours] [Maximum Marks:100

Note: Answer **any five** question. All Question carry equal marks.

1. (a) State and prove fundamental theorem of finite difference.
   (b) Express $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$ in tern of the factorial notation.
2. (a) Calculate the value of
   (i) $\Delta^2 O^5$
   (ii) $\Delta^5 O^5$
   (b) Prove that if m is positive integer then
   (i) $\Delta\, ^2x^{(m)} = m(m-1)x^{(m-2)}$
   (ii) $x^{(m+1)} + mx^{(m)} = x.x^{(m)}$
3. (a) Solve following difference equation.
   $a_r - 4a_{r-1} + 4a_{r+2} = 0, a_0 = 1, a_1 = 3$
   (b) Solve following difference equation by matrix method
   $a_{r+2} - a_r = 0, a_0 = 1, a_1 = 2$
4. (a) Solve following difference equation by using generating function
   $a_{r-2}a_{r-1} + a_{r-2} = 2^r, r \geq 0$
   Given that $a_0 = 2, a_1 = 1$
   (b) Using 2-transform solve the equation
   $a_r - 3a_{r-1} - 4a_{r-2} = 8r + 6$
5. Explain the following-
   (i) Green function
   (ii) Disconjugacy
   (iii) Sturmuian theory
   (iv) Chaotic behaviour
6. (a) State and prove fundamental lemma of calculus of variations.
   (b) Derive Euler's equation for extremely solution of a function.
7. (a) Explain the asymptotic behaviour of solution of perturbed equation.
   (b) Find non-trivial solution of the Sturm-Liouville provelem
   $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = 0, y'\left(\frac{\pi}{2}\right) = 0$
8. (a) State and solve Brachistochrone Problem.
   (b) Solve following functional
   $I[y] = \int_0^4 [xy' - y'^2]dx,\ y(0) - 0, y(4) = 3$
9. (a) State and prove the Sturm separation theorem.
   (b) If $\overline{L_n}$ is the adjoint operator of L_n then L_n is the adjoint operator of $\overline{L_n}$.
10. for the system of equations
    $\frac{dx}{dt} = 2x - y + 2xy$
    $\frac{dy}{dt} = x - 2y - xy$
    Verify that (0,0) is a critical point show that the system is almost linear and discuss the type and stability of the critical point (0,0)