



AI-1549

M. A. / M. Sc. (Final)

Term End Examination, 2020-21

MATHEMATICS

Paper : Fifth

(Difference Equation)

Time Allowed : Three hours.

Maximum Marks : 100

Minimum Pass Marks : 36

Note : Answer any **five** questions. All questions carry equal marks.

1. (a) Express  $f(x) = 3x^4 - 4x^3 + 6x^2 + 2x + 1$  terms of factorial notations.

(b) Prove that :

$$u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + \frac{u_3 x^3}{3!} + \dots = e^x \left[ u_0 + x \Delta u_0 + \frac{x^2}{2!} \Delta^2 u_0 + \dots \right]$$

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2. (a) Define generating functions, solve :

$$y_{x+1} = -y_x + 1, x = 0, 1, 2, 3, \dots, \text{ when } y_0 = 1$$

(b) Obtain solution of the difference equation of the type :

$$y_{x+1} = A y_x + B$$

3. (a) Solve :

$$y_{x+4} - 4y_{x+3} + 8y_{x+2} - 8y_{x+1} + 4y_x = 0$$

(b) Solve :

$$y_{x+2} - 4y_{x+1} + 4y_x = 3x + 2^x$$

4. (a) Find the Z-transform of  $\cosh \frac{n\pi}{2} + a$

(b) If  $Z[\langle f(n) \rangle] = F(z)$ , then prove that :

$$Z = \left[ \left\langle \sum_{k=-\infty}^n f(k) \right\rangle \right] = \frac{F(z)}{1 - z^{-1}}$$

5. (a) Solve the difference equation

$$f_{n-3} - 3f_{n-2} + 3f_{n-1} - f_n = F(n)$$

with  $f(0) = f(1) = f(2) = 0$  by Z-transforms.

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(b) Determine the type and stability of the critical point  $(0, 0)$  of the almost linear system

$$\begin{aligned}\frac{dx}{dt} &= 4x + 2y + 2x^2 - 3y^2 \\ \frac{dy}{dt} &= 4x - 3y + 7xy\end{aligned}$$

Also, find the general solution of the corresponding linear system.

6. (a) Define self-adjoint. State and prove the necessary and sufficient condition for the second order linear differential equation

$$a_0(x) \frac{d^2y}{dx^2} + a_1(x) \frac{dy}{dx} + a_2(x)y = 0$$

where  $a_0(x) \neq 0$  and  $a_0''(x)$ ,  $a_1'(x)$  and  $a_2(x)$  are continuous on  $a \leq x \leq b$ .

(b) Find the extremal of functional  $I[y(x)]$  where

$$I[y(x)] = \int_0^{\log 2} (e^{-x} y^2 - e^x y^2) dx$$

7. (a) Obtain non-trivial solution of the Sturm-Liouville problem

$$\frac{d^2y}{dx^2} + \lambda y = 0 \text{ with } y(0) = 0, y\left(\frac{\pi}{2}\right) = 0$$

(b) Show that the shortest curve joining two fixed points is a straight line.

8. (a) If  $\langle f(n) \rangle = F(z)$  where

$$F(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$$

evaluate the value of  $f_2$  and  $f_3$ .

(b) State and prove the Sturm separation theorem.

9. (a) If the second order difference equation is written in the form of  $a_{n+2} - Ba_{n+1} + Ca_n = 0$ , where B and C are constants, show that the stability condition may be written as

$$-1 - C < B < 1 + C, C < 1$$

(b) Obtain the necessary condition for

$$\int_{x_1}^{x_2} f(x, y, y') dx$$

to be an extremum.

10. (a) Find the plane curve of fixed perimeter and maximum area.

(b) Explain boundary value problems for differential equations.