PC-256
(512) M.A./M.Sc. Mathematics (SECOND SEMESTER)
Examination- JUNE-2020

Compulsory/Optional

Group -

Paper-IV

COMPLEX ANALYSIS (II)

Time:- Three Hours] **[Maximum Marks:80**

नोट : दोनो खण्डों से निर्देशानुसार उत्तर दीजिए। प्रश्नों के अंक उनके दाहिनी ओर अंकित है।

Note: Answer from Both the Section as Directed. The Figures in the right-hand margin indicated marks.

Section-A

1. Answer the following question: **1x10**

(a) Write approximate value of Euler's constant.

(b) The Riemann's functional equation $\varphi(z)$ is defined for -------------.

(c) The radius of convergence of series

$$\sum_{n=0}^{\infty} \frac{z^n}{z^{n+1}} \text{ is } \text{---------------------}$$

(d) The order of $2Z^3+5Z^2+1$ is 2 (True/False)

(e) The order of e^z is 1 (True/False)

(f) If g(z) is a polynomial of order 2 then the order of $e^{g(z)}$ is --------.

(g) If f(z) is an entire function that omits two values then f(z) is ----------.

(h) The Bloch's constant B is defined for ----------------

(i) The Landav's constant L is defined for --------------.

(j) If the exponent of convergence of zeros of canonical product is n then the order of this canonical product is -----------.

2. Answer the following question: **2x5**

(a) Prove that

$$\sqrt{z+1} = z\sqrt{z}, z \neq 0, -1, -2 \text{ ----------}$$

(b) Prove that analytic continuation of an analytic function is unique .

(c) Find the order of f(z) = e^{e^z}.

(d) Define canonical product of finite order.

(e) Prove that the inverse of Univalent function is Univalent.

Section-B

Answer any five the following question: **5x12**

3. If $|z| \leq 1$ *and* $P \geq 0$ *then* $\left|1 - E_{p(z)}\right| \leq |z|^{p+1}$

OR

State and prove Mittag-Leffler theorem.

4. State and prove Schwarz's Reflation Principle.

OR

(a) Find the analytic continuation of the function

$f(z) = \int_0^{\infty} t^2 \, e^{-zt} \, dt$

(b) Use the Schwarz's Reflection Principle prove that

$\overline{\sin z + \cos z} = \sin \bar{z} + \cos \bar{z}$

5. (a) State and prove Harnack's Inequality.

(b) Prove that

$$\sin hz = \prod_{n=1}^{\infty}(1 + \frac{z^2}{n^2\, z^2})$$

OR

Let P(z) be a canonical product of finite order p and q>o and ∈>o then for all sufficiently large $|z|$

$$\log|P(z)| > -|z|^{p+\epsilon}$$

6. State and prove Bloch's theorem.

OR

State and prove Schottky's theorem

7. Let f be on entire function of genus μ then prove that for each positive number α there exist a number r_o such that

$|f(z)| < \exp(\alpha|z|^{\mu+1}),\ |z| > r_o$

OR

(a) Use Hada Mand's factorization theorem to show that

$$\text{Sim}\ \pi z = \pi z \prod_{n=0}^{\infty}\left(1 - \frac{z^2}{n^2}\right)$$

(b) Let g be analytic is B(O,R), g(O) = 0,

$|g'(o)| = \mu > o$ and $|g(z)| \leq M$ for all z

Then prove that :

$g[B(O;R)] \supset B(O; \frac{R^2\mu^2}{6M})$