

PD-256  
M.A./M.Sc. Mathematics (SECOND SEMESTER)  
Examination- JUNE-2021  
Compulsory/Optional  
Group -  
Paper-IV  
COMPLEX ANALYSIS (II)

Time:- Three Hours ]

[Maximum Marks:80

नोट : दोनों खण्डों से निर्देशानुसार उत्तर दीजिए। प्रश्नों के अंक उनके दाहिनी ओर अंकित हैं।

Note: Answer from Both the Section as Directed. The Figures in the right-hand margin indicated marks.

Section-A

1. Answer the following question:

1x10

- a. Wallis formula is-----
- b. What is the Riemann zeta function?
- c. Define function element of  $z$ .
- d. What is Natural boundary?
- e. Define Harmonic conjugate.
- f. What is poisson kernel?
- g. Define convex function.
- h. Write standard form for an entire function.
- i. Define Bloch's constant.
- j. Write univalent function.

2. Answer the following question:

2x5

- (a) If  $|z| \leq 1$  and  $P \geq 0$ . Then  $|1 - E_p(z)| \leq |z|^{P+1}$  when  $E_p(z)$  is elementary factor.
- (b) Find the radius of convergence for  $\sum_{n=1}^{\infty} \left( \frac{z^n}{2^{n+1}} \right)$
- (c) Write Dirichlet Region.
- (d) Find the order of the function  $\cos Z$  and  $\sin Z$
- (e) Write the statement of  $\frac{1}{4}$  - Theorem.

Section-B

Answer any five the following question:

5x12

3. State & prove Legendre's duplication formula.
4. State & prove Euler's product formula.
5. State & prove Schwarz's reflection principle for symmetric region.
6. state & prove monodromy theorem.
7. state & prove Harnack's Inequality.
8. Let  $G$  and  $\Omega$  be regions such that there is a one-one analytic function of  $G$  on to  $\Omega$ . Let  $a \in G$  and  $\varphi = F(a)$ . If  $g_a$  and  $\varphi$  are the Green's functions for  $G$  and  $\Omega$  with singularities  $a$  and respectively, then  $G_a(z) = \varphi(f(z))$
9. state & prove Jensen's formula.
10. state & prove Hadamard's three circles theorem.
11. state & prove Bloch's theorem.
12. state & prove Schottky's theorem.