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AI -1541

M. A./M. Sc. (Privious)
Term End Examination, 2020-21

MATHEMATICS

Paper : Third

(Topology)

Time Allowed : Three hours

Maximum Marks : 100

Minimum Pass Marks : 36

Note : Answer any five questions. All questions carry equal marks.

1. (a) Let $\{\mathfrak{J}_\lambda : \lambda \in \wedge\}$ where $\wedge$ is an arbitrary set, be a collection of topologies for *X*. Then prove that the intersection $\bigcap\{\mathfrak{J}_\lambda : \lambda \in \wedge\}$ is also a topology for *X*.

(b) In a topological space *X* prove that a subset *A* of *X* is closed iff $D(A) \subset A$.

2. (a) Show that a topological space *X* is disconnected if and only if there exists a non empty proper subset of *X* which is both open and closed in *X*.

(b) Show that a subset of a topological space is open if and only if it is the neighbourhood of each of its points.

3. (a) Let *Y* be a subspace of a topological space *X*. If $A \subset Y$ is open (closed) in *X*, than *A* is also open (closed) in *Y*.

(b) Let *X* and *Y* be topological spaces. A mapping $f : X \to Y$ is continuous if and only if the inverse image under *f* of every closed set in *Y* is closed in *X*.

4. (a) Show that two closed (open) subsets *A* and *B* of a topological space are separated if and only if they are disjoint.

(b) Show that continuous image of a connected space is connected.

5. (a) Let Y be a subspace of a topological space X and let $A \subset Y$. Then show that A is compact relative to X if and only if A is compact relative to Y.

(b) Show that closed subsets of compact sets are compact.

6. (a) Show that every closed subspace of a Lindelof space is Lindelof.

(b) Show that every T_3 space is T_2 space.

7. State and prove Tietze extension theorem.

8. Let $(X, \mathfrak{J})$ be a topological space and let $Y \subset X$. Then, show that Y is $\mathfrak{J}$-open iff no net in $X - Y$ can converge to a point.

9. State and prove Tichonoff theorem.

10. Prove that, every filter on a set X is contained in an ultrafilter on X.