

AH – 1541 CV-19
M.A./M.Sc. (Previous)
Term End Examination 2019-20
MATHEMATICS
Paper – III
Topology

Time : Three Hours]

[Maximum Marks : 100

[Minimum Pass Marks : 36

Note : Answer any five question. All question carry equal marks.

1. (a) let $\{J_\lambda : \lambda \in \Delta\}$ Where Δ is an arbitrary set, be a collection of topological Space for X . Then prove that $\cap \{J_\lambda : \lambda \in \Delta\}$ is also a topology for X .
(b) Given an example of a topological Space different from the discrete and indiscrete Space in which open Sets are exactly the same as closed Sets.
2. (a) Prove that a subset A of a metric Space X is closed if and only if. A Contains all its limit Points.
(b) Let $X = \{a, b, c\}$ and $J = \{\emptyset, X, \{b\}, \{a, c\}\}$ find the interior, closure and Set of all cluster points of the set $\{a, b\}$.
3. (a) Show that a subspace of a topological Space is it Self a topological Space.
(b) Let A be uncountable Subset of Space whose topology has countable base. Then Prove that some Point of A is a Limiting Point of A .
4. (a) Let X , and Y be topological Spaces. Prove that a mapping $f : X \rightarrow Y$ is continuous if and only if the inverse image under f of every open set in Y is open in X .
(b) Prove that Homeomorphism is an equivalence relation in the class of topological and Space.
5. (a) Prove that a topological Space (X, J) is disconnected if and only if \exists a non empty proper subset of X Which is both J -Open and J -closed in X .
(b) Prove that the Set of real number With usual topology is Connected.
6. (a) Prove that the closure of connected Set is connected.
(b) Prove that every closed Subspace of compact Space is compact.

7. (a) Prove that a Continuous image of a compact Space is Compact.
- (b) Stat and Prove compact Space have Bolzano Weierstrass Property.
8. (a) Stat and prove Le besgue-covering Lemma.
- (b) Prove that a topological Space (X, J) is a J - Space if and only if every Singleton Set $\{x\} \in X$ is closed.
9. State and Prove Tietze- extension theorem.
10. (a) State and Prove the Stone - compactification Theorem.
- (b) Prove that every filter is contained in an ultra filter.

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