AI-1540 CV-19

M.A./M.Sc. (Previous)

Term End Examination, 2020-21

MATHEMATICS

(Real Analysis and Measure Theory)

Paper - II

Time : Three Hours] **[Maximum Marks : 100**

[Minimum Pass Marks : 36

Note : Answer any five questions. All questions carry equal marks.

1. (a) If the $\lim\limits_{||p||\to 0} s(p, f, \alpha)$ exists, then prove that
$f \in R(\alpha)$ and $\lim\limits_{||p||\to 0} s(p, f, \alpha) = \int_a^b f d\alpha$
(b) Let $\alpha(x) = |x|^3$ then find the value of $\int_{-1}^{2} x^5 \, d\alpha$

2. (a) Prove that the sum of an absolute convergent series does not alter with any rearrangement of terms.
(b) Let y : $[a, b] \to R^k$ be a curve, If $c \in (a, b)$ Then prove that-
$\hat{}\tau(a, b) = \hat{}\tau(a, c) + \hat{}\tau(c, b)$

3. (a) State and prove cavchy General principle of uniform convergence.
(b) Show that the sequence {fn} where $fn(x) = \frac{x}{1+nx^2}$
Converges uniformly on R

4. (a) Let {fn} be a sequence of real valued function on a metric space (x, d) which converges uniformly to the function f on x. If each fn (n=1; 2, 3 ---) is continuous on x then f is also continuous on x.
(b) Test for uniform convergence and term by term integration of series $\Sigma = \frac{x}{(n+x^2)^2}$

5. (a) Find the radius of convergence of the power series.
(i) $\sum\limits_{n>1}^{\infty} \frac{3^n}{\sqrt{n+1}}$ (ii) $\sum\limits_{n>1}^{\infty} \frac{n!}{n!} 3^n$
(b) state and prove Tauber's Theorem.

6. State and prove Inverse function Theorem.

7. (a) find the shortest distance from the point (3/2, 0) to the parabola $y^2 = yx$
(b) State and prove chain rule.

8. (a) Let $\{ E_n\}$ be a countable collection of sets of real numbers then prove that
$m^*(\bigcup\limits_{n=1}^{\infty} An) \leq \sum\limits_{n=1}^{\infty} m^*(\in n)$
(b) Prove that a continuous function defined on a measurable set is measurable.

9. (a) State and prove Bounded convergence theorem.
(b) Let f be a bounded function defined in [a, b] If f is Riemann integrable over [a, b], then it is lebesgue integrable and
$R\int_a^b f(x)dx = \int_a^b f(x)dx$

10. (a) If f is absolutely continuous on [a, b] then prove that f is of bounded variation.
(b) Let $1 \leq p \leq \infty$ and let $f, g \in l^p(\mu)$, then $f+g \in l^p(\mu)$ and $||f+g||_p \leq ||f||_p + ||g||_p$