Printed Pages – 4

AI-1542

M. A./M. Sc. (Previous)
Term End Examination, 2020-21

MATHEMATICS

Paper : Fourth

(Complex Analysis)

Time Allowed : Three hours

Maximum Marks : 100

Minimum Pass Marks : 36

Note : *Answer any **five** questions. Answer to each question should begin on a fresh page. All questions carry equal marks.*

1. (a) State and prove cauchy's integral formula.

(b) Obtain the Taylor's series and Laurent series which represent the function :

$$\frac{Z^2-1}{(Z+2)(Z+3)}$$ in the regions :

(i) $2<|z|<3$

(ii) $|z|>3$

(iii) $|z|<2$

2. (a) State and prove argument principle.

(b) Prove that :

$$\cosh\left(z+\frac{1}{z}\right)=a_0+\sum a_n\left(z^n+\frac{1}{z^n}\right)$$

where $a_n=\frac{1}{2z}\int_0^{2z}\cosh(2\cos\theta)\cos n\theta\,d\theta$.

3. (a) State and prove Rouche's Theorem.

(b) Prove that all the roots of $2^7-5z^3+12=0$ lie between the circles $|z|=1$ and $|z|=2$.

4. (a) (i) Find the singularities of :

$$f(z)=\frac{z-2}{z^2}\sin\left(\frac{1}{z-1}\right)$$

(ii) Find the residue of $\frac{z^3}{z^2-1}$ at $z=\infty$

(b) Apply calculus of residue to prove that

$$\int_0^{\infty}\frac{dx}{1+x^2}=\frac{\pi}{2}$$

5. (a) Write all the critical points and fixed points of bilinear transformation :

$$W=\frac{az+b}{cd+d}\,(ad-bc\neq 0)$$

(b) If $w=f(z)$ represents a conformal mapping of a domain D in the z plane into a domain D' at the w plane the $f(z)$ is an analytic function at z in D'.

6. (a) State and prove Hurwitz theorem.

(b) If $\mathrm{Re}(z)>1$ then prove that :

$$\int_0^{\infty}\left(e^t-1\right)^{-1}t^{z-1}\,dt=\zeta(z)\,\Gamma z$$

7. State and prove Weirestrass factorisation theorem.

8. (a) State and prove Runge's theorem.

(b) State and prove Bloch's theorem.

9. (a) State and prove Hadamard's three circle theorem.

(b) State and prove Shwartz's Reflection principle.

10. (a) State and prove Harnack's inequality.

(b) State and prove Mittag-Lefflex's Theorem.