AI -1544

M. A./M. Sc. (Previous)
Term End Examination, 2020-21

MATHEMATICS

Paper : Fifth

(Advanced Discrete Mathematics)

Time Allowed : Three hours

Maximum Marks : 100

Minimum Pass Marks : 36

*Note : Answer any **five** questions. Answer to each question should begin on a fresh page. All questions carry equal marks.*

1. (a) Define and explain each of the following :
   (i) Conditional and Biconditional statements
   (ii) Converse, inverse and contrapositive of $p \to q$.

   (iii) Equivalent statement.
   (iv) De Morgan's laws.

   (b) Prove that :

$$\sim(p \wedge q) \to (\sim p \vee (\sim p \vee q))$$

   without constructing truth table.

2. (a) Define arguments. Modus ponens and law of syllogism.
   Prove that the following arguments is valid :

$$\begin{array}{l} p \\ p \to q \\ \hline q \end{array}$$

   (b) (i) Simplify the following :
   (I) $(P \wedge Q) \wedge \sim P$
   (II) $\sim(\sim P \wedge Q) \wedge (\sim P \vee Q) \wedge (P \vee Q)$

   (ii) Using $\wedge$ and $\sim$ for A and N respectively rewrite the following statements :
   (I) A N A P A q p A A N q r p
   (II) A A p N r A q N p

3. (a) Define idempotent element of a semi group. Prove that every finite semigroup has an idempotent element.

(b) Define Homomorphism of Monoids. Let (S, $*$) and (T, o) be semigroups. If $f : S \to T$ is a semigroup homorphism, then semi group (T, o) is isomorphic to some quotient semigroup of (S, $*$)

4. (a) Establish the equivalence of the two definitions of a lattice.

(b) Explain Bounded lattices. Prove that every finite lattice is a bounded lattice.

5. (a) In any boolean Algebra, show that :

(i) $(a+b)(b+c)(c+a) = ab+bc+ca$

(ii) $(a+b')(b+c')(c+a') = (a'+b)(b'+c)(c'+a)$

(b) (i) Express the following Boolean functions

$$f(x,y,z) = (x+y)(x+y')(x'+z)$$

in disjunctive normal form in three variables.

(ii) In a Boolean Algebra, show that :

$$f(x,y) = xf(1,y) + x'f(0,y)$$

6. (a) Use the Karnaugh map respresentation to find a minimal form of each of the following functions :

(i) $f(x,y) = x'y + xy$

(ii) $f(x,y,z) = xyz + xy'z + x'yz + x'y'z$

(b) (i) Draw the logic circuit for the following expression :

$$x \cdot y + zy'$$

(ii) Draw the logic circuit with inputs a, b, c and output f where

$$f = ab'c + abc' + ab'c'$$

7. (a) Explain phrase structure grammer. Find the language $L(G)$ over $A = \{a, b, c\}$ genereated by the grammer G with production

$$S \to aSb,\ aS \to Aa,\ Aab \to C$$

(b) What is polish notation. Explain conversion of infix experience to polish notation.

8. (a) Design a finite state machine M which can add two binary numbers.

(b) Define equivalent machine. Construct the state diagram for the finite state machine with the state table as given below :

State	*f* input		*g* output	
	0	1	0	1
S_0	S_1	S_0	1	0
S_1	S_2	S_1	0	1
S_2	S_3	S_1	1	1
S_3	S_2	S_1	0	0

9. (a) Define the following :

(i) Adjacent vertex

(ii) Degree vertex

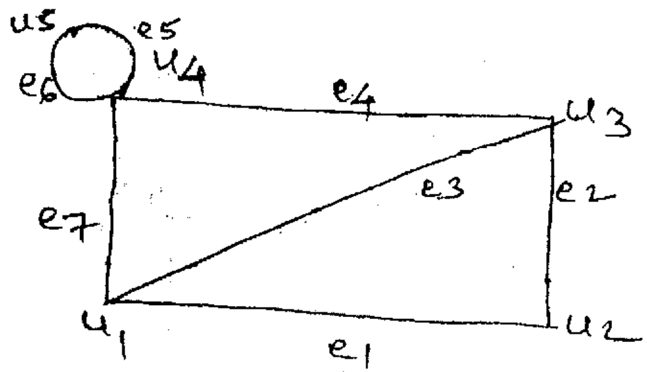
(iii) Isolated vertex

(iv) Pendent vertex

(v) Regular graphs

(b) Define In-degree and out-degree, Adjacency matrix, Incidence matrix. Write the incidence

matrix of the following graph :



10. (a) Define spanning tree with example. Define Branch and chord of a spanning tree. Prove that every connected graph has atleast one spanning tree.

(b) Define cut set with example. Write properties of cut sets. If every region of a simple planer graph with n vertices and e edges is bounded by k edges, show that :

$$e = \frac{k(n-2)}{(k-2)}$$