AI-1539
M.A./M.Sc. (Previous)
Mar.-Apr. 2021
Compulsory/Optional
Group- MATHEMATICS
Paper-

Name/Title of Paper- ADVANCED ABSTRACT ALGEBRA
Time: 3:00 Hrs.]

[Maximum Marks: 100
[Minimum Pass Marks: 36

Note: **Attempt any five questions. All questions carry equal marks.**

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1. a. Let G be a group and let G' be the derived group of G' then show that
   i. G/G' is abelian
   ii. If $H \leq G$ Then G/H is abelian if and only if $G' \leq H$.

   b. Show that every important group is solvable.

2. a. Let H and K be distinct maximal normal Subgroup of G. Then Show that $H \cap K$ is a Maximal normal Subgroup of H and also of K.

   b. Show the a simple group is Soluble if and only if it is cyclic.

3. a. Give an example of a non-abelian group each of whose Subgroup is normal.

   b. If G is a Cyclic group such that $|G|=P_1P_2 \ldots\ldots\ldots P_r P_i$ distinct Primes, Show That the number of distinct composition series of G is r!

4. a. Let N be a normal Subgroup of the group G. Then Show that G/N is a group under multiplication, The mapping
   $\emptyset = G \rightarrow G$ *Given by* $x \rightarrow xn$, *is a surjective homomorphism and* $Ker\ \emptyset = N$.

   b. Let Show that a group of order P^n (P Prime) is impotent.

5. a. Show that In a nonzero Commutative ring with unit and ideal M is maximal if and only if R/M' is a field.

   b. Let A and B be two mxn Matrices over a field F. Show that $rank\ (A + B) \leq rankA + rankB$.

6. a. Let R be a Commutative ring with unity in which each ideal is prime then show that R is a field.

   b. Show that the Sub modules of the quotient module M/N are of the form U/N, Where U is a Sub module of M Containing N.

7. a. Let P(x) be an irreducible polynomial if F [x]. Then Show That There exists an extension E of F in Which P(x) has a root.

   b. Is $R\sqrt{-5}$ *normal over* R?

8. a. Prove that $\sqrt{2}$ *and* $\sqrt{3}$ *are algebraic over Q. find the degree of* $Q(\sqrt{2} + \sqrt{3})$ *over* Q.

   b. Let F be a finite field. Then Show that there exists an irreducible polynomial of any given degree n over F.

9. a. Let A be a minima left ideal in a ring R. Then show that either $A^2=(O)$ or A=Re, There is an idempotent in R.

b. Let H be a finite Subgroup of the group of automorphism of field E, Then Show that

$$[E:E_H]=[H]$$

10. a. Let N be a nil ideal in a noetherian ring R, Then Show that N is idempotent.

b. Show that the Polynomial $x^7-10x^5+15x+5$ is not Solvable by radical over Q.