PD-155-S.E.-CV-19

M.A./M.Sc. MATHEMATICS (1st Semester)

Examination, Dec.-2020

Paper-

TOPOLOGY (I)

Time : Three Hours] [Maximum Marks : 80

[Minimum Pass Marks : 29

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Answer the following questions:- 1x10=10
   (a) Define discrete topology.
   (b) If F_1 and F_2 be two closed subsets of a topological space X, the show that $F_1 \cup F_2$ is a closed set.
   (c) Define sub-base for a topology.
   (d) Define closure of a set.
   (e) Define homeomorphism in topological spaces.
   (f) Define Normal space.
   (g) Show that every discrete space is a T_0 space.
   (h) If (R, U) is usual topology, write interior of $A = (0.1)$
   (i) Let $X = \{a, b, c\}$ and $T = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ is topology on write one neighborhood of $A = \{a, c\}$
   (j) Define completely regular space.
2. Answer the following questions:- 2x5=10
   (a) Let (X, T) be a topological space and let A,B be any two subsets of X, then show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
   (b) Write Kuratowski closure axioms.
   (c) Let $X = \{a, b, c\}$ and $T = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ is a topology on X. Find the limit points of the set $A = \{a, c\}$.
   (d) Show that every metric space is a T_2 space.
   (e) Show that the property of being T_0 space is hereditary.

Section-B

12x5=60

Answer all questions.

3. Let (X, T) be a topological space and $A \subset X$ then prove that
   $\overline{A} = \{y \in X: \text{every nbd of y meets A non} - \text{vacously}\}$

   OR

   Let A and B be any two subsets of topological space (X, T) then prove the following
   (i) $D(\emptyset) = \emptyset$
   (ii) $A \subset B \Rightarrow D(A) \subset D(B)$
   (iii) $D(A \cap B) \subset D(A) \cap D(B)$
   (iv) $D(A \cup B) = D(A) \cup D(B)$
4. (a) Show that a constant function is continuous.
   (b) Show that $f: (X, \mathcal{T}) \to (Y, \mathcal{U})$ is continuous if and only if
   $f[\overline{A}] \subset \overline{f[A]}$ - $\forall A \subset X$

   OR

   (a) Show that compositions of continuous functions are continuous.
   onto
   (b) Show that $f:(X,\mathcal{T}) \to (Y,\mathcal{U})$ is a homeomorphism only if

   $f[\overline{A}] = \overline{f[A]}$ $\forall A \subset X$

5. **Define second countable space. Show that a homeomorphic image of a second countable space is second countable.**

OR

Prove that every regular Lindelof space is normal.

6. **Show that every convergent sequence in Hausdorff space has a unique limit.**

OR

Show that normality of a space is a topological property.

7. **Show that every second countable space is separable**

OR

Show that the space (R, U) is T_3 space.