PD-154-S.E.-CV-19

M.A./M.Sc. MATHEMATICS (1st Semester)

Examination, Dec.-2020

REAL ANALYSIS (I)

Time : Three Hours] **[Maximum Marks : 80**

[Minimum Pass Marks : 29

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. **Answer the following questions:-** **1x10=10**

(a) **Write one vefinement partitions**

$$P = \{2,\ 2.5,\ 3.1,\ 3.7,\ 4\}$$

(b) **If P_1 and P_2 are two partion of interval $[a,\ b]$ α is bounded and monotonic increasing function on $[a,\ b]$ then write the relation between lower RS-Sum of P_1 and upper RS-Sum of P_2**

(c) **Let $f(x) = x,\ g(x) = x^2,\ Does\ \int_0^1 fdg\ exist$?**

(d) **Write the relation between the Riemann integral and the Riemann-stielfjes integral.**

(e) **Does sequence $f_n: [0,\ 1] \to R$**

$f_n(x) = x^n,\ \ x \in [0,\ 1]$ **converges pointwise?**

(f) **Every pointwise convergent series is uniform convergent. (True/False)**

(g) **The condition of uniform convergence of series is necessary condition for the validity of term by term integration of that series. (True/False)**

(h) **Define contraction mapping.**

(i) **Define invertible Linear operators.**

(j) **Write the radius of convergence of series**

$$\sum \frac{nx^n}{(n+1)^2}$$

2. **Answer the following questions:-** **2x5=10**

(a) **Show that**

$$\int_0^2 [x]d(x^2) = 3$$

(b) **Obtain first two Bernstein polynomials for**

$$f(x) = e^x, \quad x \in [-1,\ 1]$$

(c) **For the series of functions $\sum \frac{x}{(n+x^2)^2}$ find a convergent series M_n with**

$$\left|\frac{x}{(n+x^2)^2}\right| \le M_n,\ \ \forall n$$

(d) **Prove that contraction map is always continuous.**

(e) **Find the radius of convergence of the power series**

$$\sum_{n=1}^{\infty} \frac{Z^n}{(n+1)^B} \qquad (B > 1)$$

Section-B **12x5=60**

Answer all questions.

3. (a) **Let f be continuous and α monotonically increasing on $[a,\ b]$ then $f \in R(\alpha)$ On $[a,\ b]$**

(b) **If the $\text{Lim}_{\|P\|\to 0} S(P, f, \alpha)$ exists then $f \in R(\alpha)$ and $\text{Lim}_{\|P\|\to 0} S(P, f, \alpha) = \int_a^b f d\alpha$**

OR

State and prove Implicit function theorem.

4. **If Y' is continuous on $[a,\ b]$ then prove that Y is rectifiable and**

$$\Lambda(Y) = \int_a^b |Y'(t)|dt.$$

OR

State and prove Tauber's theorem.

5. (a) State and prove Cauchy's general principle of uniform convergence.
   (b) Prove that sequence $\left\{\frac{nx}{1+n^2x^2}\right\}_{n=1}^{\infty}$ does not converges uniformly on R.

OR

   (a) Verify the series $\sum_{n=0}^{\infty} x^n$, $|x| < 1$ for term by term differentiation.
   (b) Prove that the series $\sum_{n=1}^{\infty} \frac{n\sqrt{x}}{1+n^2x^3}$ can not integrated term by term in $[0, \infty]$

6. Prove that Limit function of a uniformly convergent sequence of continuous functions is continuous but converse is not True.

OR

   (a) State and prove Chain rule.
   (b) Let f maps a convex open set $E \subset R^N$ into R^M, f be differentiable in E and $\ni$ A real number M such that

$$\|f'(x)\| \leq M, \qquad \forall x \in E$$

   Then prove that

$$|f(b) - f(a)| \leq M|b - a|, \quad \forall\, a, b \in E$$

7. State and prove Inverse function theorem.

OR

   If $\vec{f}$ maps $[a,\ b]$ into R^k and if $\vec{f} \in R(\alpha)$ for some monotonically increasing function α on $[a,\ b]$ then prove that $|\vec{f}| \in R(\alpha)$ and

$$\left|\int_a^b \vec{f}\, d\alpha\right| \leq \int_a^b |\vec{f}|\, d\alpha$$