PC - 257

(512) M.A/M.Sc. MATHEMATICS (1ST SEMESTER)

Examination JUNE 2020

Compulsory/Optional

Group-

Paper-V

ADVANCED DISCRETE MATHEMATICS (II)

Time:- Three Hours | Maximum Marks: 080

Minimum Passing Mark- 29

नोट : दोनों खण्डों से निर्देशानुसार उत्तर दीजिए। प्रश्नों के अंक उनके दाहिनी ओर अंकित हैं।

Note: Answer from Both the Section as Directed. The Figures in the right-hand margin indicate marks.

Section - A

1. Answer the following questions: 2X10

(a) Define type -2 grammer.

(b) Define context-free grammers.

(c) A directed graph, called a ..... is associated with a finite automation. By a 5-type $(Q, \sum, \delta, 90, F)$

(d) Define information-processing machine.

(e) Define equivalent machine.

(f) A vertex v of a connected graph $G = (V, E)$ is called a cut point if its deletion makes the remaining graph as .....

(g) An edge is said to be a ..... If it's both end vertices are same.

(i) Define homeomorphic graphs.

(j) A tree T with n vertices has ..... edges.

2. Answer the following short-answer type questions : -

(a) Consider the grammer G = (N,T,P,S) where P = (S→aA, S→b, A→aa) with S as start symbol. Find L(G)

(b) Define Moore machine.

(c) What is minimization of a machine.

(d) Define Hamiltonian circuit Hamiltonian path, and Hamiltonian graph.

(e) Explain directed graph.

Section - B

Answer the following questions : 12X5

3. (a) Show that the language $L(G) = \{a^n b^n c^n : n \geq 1\}$ can be generated by G = (N,T,P,S) where N (S,B,C), T = (a,b,c), P = (S→aSBc, S→aBc, cb→Bc, aB→ab, bB→bb, bc→bc, cc→cc)

(b) Use the grammer G given as

G = C{{S,A,B},{a,b},P,S}

Where P = {(CS→AB), (S→bA), (A→a), (A→aS), (A→bAA), (B→b), (B→bS), (B→aBB)}

To construct the derivation tree for the strings – (i) aaabbb (ii) abababba

OR

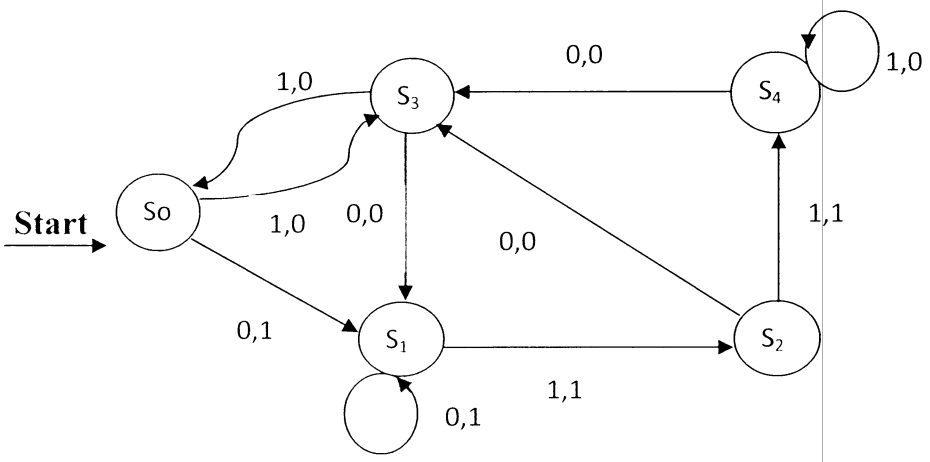
(a) Write Notes an Polish Notation.

(b) Consider the Moore machine described by the transition table in following table. Construct the corresponding Mealay Machine.

Present State	Next State		Output
	a = 0	a = 1	
S_1	S_1	S_2	0
S_2	S_1	S_3	0
S_3	S_1	S_3	1

4. (a) Design a finite state machine that receives the set {0,1,2} as input and produces an output such that output is equal to the madulo 3 sum of digits in the input sequence.

(b) Construct the state table for the finite state machine with the diagram shown below:-



OR

(a) Minimize finite state machine M, Where M is given by the following state table

State	Input		Output
	0	1	
S_0	S_3	S_1	1
S_1	S_4	S_1	0
S_2	S_3	S_0	1
S_3	S_2	S_3	0
S_4	S_1	S_0	1

(b) Define finite state language.
Show that the language. $L = \{a^k | k = i^2, i \geq 1\}$ is not a finite state language.

5. (a) Show that the sum of the degrees of all the vertices in a graph is equal to twice the number of edges.

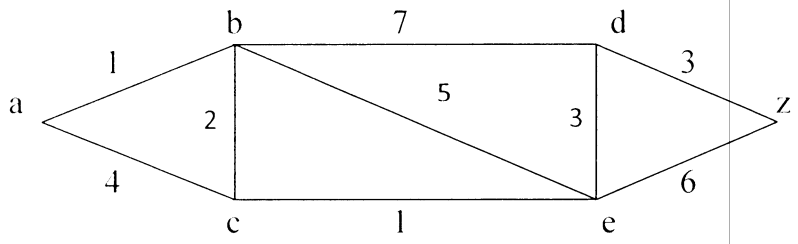
(b) Prove that if G is self-complementary then G has 4k or 4k+1 Vertices, where k is an integer.

OR

(a) Define following :-
1. Sub graph
2. Vertex disjoint sub graph
3. Edge disjoint sub graph
4. Complement of a sub graph.

(b) Define bipartite graph Show that the maximum number of edges in a complete bipartite graph of n vertices is $\frac{n^2}{4}$.

6. (a) Find the shortest path from a to z in the following graph. where numbers associated with the edges are the weights.

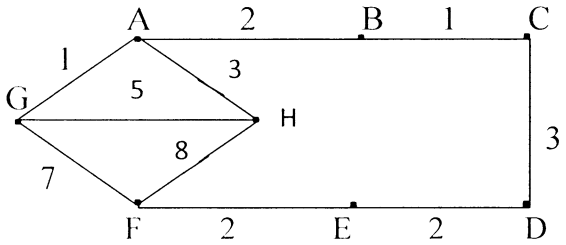


(b) Define Euler Graph.

Prove that a connected graph G is an Euler graph if and only if G is the union of some edges disjoint circuits.

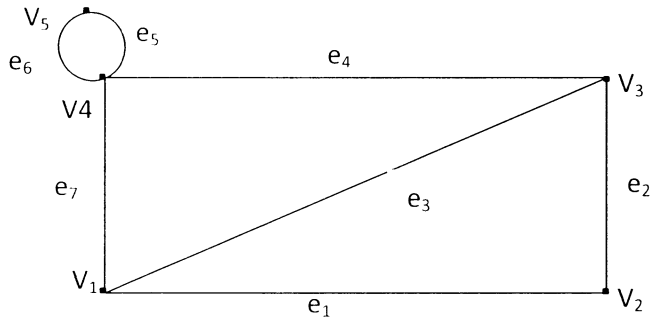
OR

(a) Solve the travelling salesman problem for the following weighted graph.



(b) Define Incidence matrix.

Write the incidence matrix of the following graph.



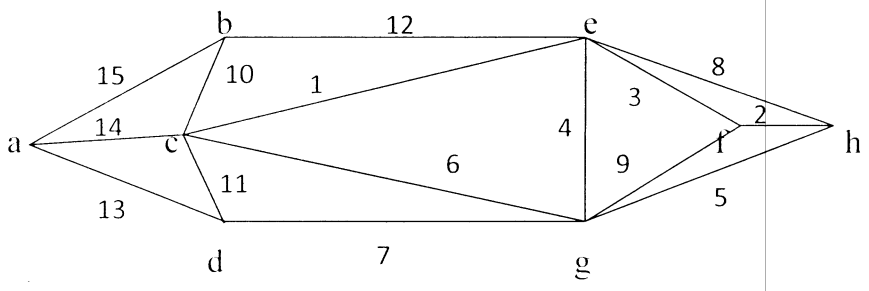
7. (a) Show that a graph with n vertices, (n-1) edges and having no circuit is connected. (ie. Graph is a tree)

(b) Define spanning tree.

Prove that every connected graph has at least one spanning tree.

OR

Find the minimum spanning tree for the following graph.



(a) by using Kruskal's Algorithm.

(b) by using prim's Algorithm.