PD-156-S.E.-CV-19

M.A./M.Sc. MATHEMATICS (1st Semester)

Examination, Dec.-2020

Paper-IV

COMPLEX ANALYSIS-I

Time : Three Hours] **[Maximum Marks : 80**

[Minimum Pass Marks : 29

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. **Answer the following questions:-** 1x10=10

(a) Who was the first which obtained the definite integral by replacing a real variable by a complex variable.

(b) If $f(z)$ is analytic within and on the boundary c of a simply connected region and $'a'$ is any point within then $\int_c \frac{f(z)}{(z-a)^3} dz = \cdots \ldots \ldots$

(c) Write the value of $\int_c \frac{z^2}{(z-a)^3} dz$ where c is circle $|z| = 3$

(d) If $f(z)$ is analytic within and on a circle c defined by $|z - z_0| = 2$ and $|f(z)| \le K$ one C then $\left|f^{(iv)}(z_0)\right| \le \cdots \ldots \ldots \ldots$

(e) Zero of an analytic functions are............

(f) A function which has no singularity.............is constant

(g) How many roots of $z^8 - 4z^4 + z^2 - 1 = 0$ lie inside the circle $|z| = 1$

(h) If $z = a$ is only singularity of an analytic function $f(z)$ inside a closed curve C then the residue of $f(z)$ at $z = a$ is..............

(i) Write the fixed point of $\frac{z-1}{z+2}$

(j) Write the condition that transformation $w = \frac{az+b}{cz+d}$ is normal.

2. **Answer the following questions:-** 2x5=10

(a) Evaluate $\int_c \frac{2z+1}{z^2+2}$ where c is $|z| = \frac{1}{2}$

(b) Expand $\frac{1}{(z-1)(z-2)}$ for the rigion $|z| > 2$

(c) Find the residue of $\frac{z^2+z+1}{z(2z+1)(z+2)}$ at $z = -\frac{1}{2}$

(d) Find the normal form of $w = \frac{z}{2-z}$

(e) Distinguish between pole and essential singularity.

Section-B

12x5=60

Answer all questions.

3. State and prove Laurent's theorem.

OR

State and prove Cauchy Goursat's theorem.

4. Find the expansion of $\frac{1}{(z^2+1)(z^2+2)}$ in power of z when

(i) $|z| < 1$ (ii) $1 < |z| < \sqrt{2}$ (iii) $|z| > \sqrt{2}$

OR

State and prove maximum modus principle.

5. Show that one root of the equation $z^4 + z + 1 = 0$ Lies in the first quadrant.

OR

If $f(z)$ is analytic within and on a closed contour C except at a finite number of poles and has no zero on C then $\frac{1}{2\pi i}\int_c \frac{f'(z)}{f(z)} dz = N - P$

Where N is the number of zero and P is the number of poles inside C (a pole or zero of order m being counted m times)

6. Find the value of following integrals by using calculus of residues

(a) $\int_0^{\pi} \frac{1+2\cos\theta}{5+3\cos\theta} d\theta$

(b) $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$

OR

Apply the method of calculus of residues to prove that

(a) $\int_0^{\infty} \frac{dx}{(1+x^2)^2} = \frac{3\pi}{8}$

(b) $\int_0^{\infty} \frac{\cos x}{x^2+a^2} dx = \frac{\pi e^{-a}}{2a}$ $\quad (a > 0)$

7. Find the bilinear transformation which transforms half plane $R(z) \geq 0$ onto the unit circular disc $|w| \leq 1$

OR

State and prove sufficient condition for $w = f(z)$ to represent a conformal mopping.