PD-157-S.E.-CV-19

M.A./M.Sc. MATHEMATICS (1^{st} Semester)

Examination, Dec.-2020

Paper-V

ADVANCED DISCRETE MATHEMATICS (I)

Time : Three Hours] **[Maximum Marks : 80**

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. **Answer the following multiple choose type questions:-** 1x10=10

   (a) Let $p \Rightarrow q$ be any conditional statement, then the inverse of $p \Rightarrow q$ is the state.

   (i) $q \rightarrow p$ (ii) $\sim p \Rightarrow \sim q$ (iii) $\sim q \rightarrow \sim p$ (iv) $p \Rightarrow \sim q$

   (b) The truth value of the following statement $7 + 2 = 9 \Leftrightarrow 6 + 3 = 8$ is:

   (i) False (ii) true (iii) False and true both (iv) None of the above

   (c) The statement $p \Leftrightarrow (\sim p)$ is.

   (i) Tautology (ii) Contradiction (iii) Logical Equivalence (iv) None of the above

   (d) Let p and q be the two statements is then the value of the statement $p \uparrow q$

   (i) $\sim(p \vee q)$ (ii) $\sim p \wedge q$ (iii) $p \wedge \sim q$ (iv) $\sim(p \wedge q)$

   (e) Which one of the following is absorption low.

   (i) $p \wedge q \equiv \sim p \vee \sim q$ (ii) $p \vee (q \cup r) \equiv (p \vee q) \vee r$

   (iii) $\sim(p \vee q) \equiv \sim p \wedge \sim q$ (iv) None of the above

   (f) In a Boolean algebra, the dual of $a \,.\, 0 = 0$ is

   (i) $a + 0 = 1$ (ii) $a + 1 = 0$ (iii) $a \,.\, 1 = 1$ (iv) $a + 1 = 1$

   (g) Let $(s, *)$ and $t, 0$ be two semigroups. A semigroup homomorphism $f : s \longrightarrow t$ is one-are also called:

   (i) Semiproup automorphism (ii) Semigroup epimorphism

   (iii) Semiproup monomorphism (iv) Semigroup isomorphism

   (h) Disjunctive normal forum of the function $x \,.\, y$ is .

   (i) $xyz + xy'z$ (ii) $xyz + x'yz$ (iii) $xyz + xyz'$ (iv) $xyx + x'y'z$

   (i) Boolean algebra deals with how many values.

   (i) it deals with only four discrete values (ii) it deals with only three discrete values

   (iii) it deals with only two discrete values (iv) it deals with only five discrete values

   (j) Complete disjunctive normal form in two variables x_1, x_2 is

   (i) $x_1x_2 + x_1'x_2'$ (ii) $x_1'x_2 + x_1x_2'$

   (iii) $x_1x_2 + x_1x_2' + x_1'x_2$ (iv) $x_1.x_2 + x_1'.x_2 + x_1.x_2' + x_1'.x_2'$

2. **Answer the following short answer type questions:-** 2x5=10

   (a) Using $\wedge$ and $\sim$ for A and N respectively rewrite the following statement.

   AApNrAqNp

   (b) Explain universal quantifier giving example.

   (c) Define complemented complete lattice.

   (d) Prove the following identity in a Boolean algebra $(B, +, \cdot, ')$

   $a.b + b.c + b.c' = b, \ \forall a, b, c \in B$

   (e) Prove that A homomorphism of from $(G,*)$ onto $(G', *)$ with Kernal k is an isomorphism if $k = \{e\}$

Section-B 12x5=60

Answer the following questions.

3. (a) Prove that $(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) \Rightarrow (p \Leftrightarrow r)$ is a tautology.

(b) Define law of Syllogism and Rule of detachment,
Show that a following argument is a valid argument.

$$p \overset{p}{\underset{q}{\wedge}} q \rightarrow \pi \vee s$$
$$\underline{\sim q}$$
$$\pi$$

OR

(a) Explain law of duality.
Express the statement $(p \vee \sim q) \rightarrow (p \wedge r)$ in terms of v and ~ only and the statement $p \wedge (q \leftrightarrow r)$ in terms $\wedge$ and ~ only.

(b) Simplify the following:-
(i) $\sim(\sim P \wedge Q) \wedge (\sim P \vee Q) \wedge (P \vee Q)$ (ii) $(P \wedge Q) \vee P$

4. (a) Find the procedure to obtain a disjunctive normal form of a given logical expression.
Obtain the disjunctive normal forms of the following
$$p \Rightarrow (p \Rightarrow q) \wedge [\sim(\sim q \vee \sim p)]$$
(b) Obtain the principle conjunctive normal form of the following
$$(\sim p \Rightarrow \pi) \wedge (q \Leftrightarrow p)$$

OR

(a) For any a and b in a Boolean algebra $(B, \vee, \wedge, ')$ show that,
(i) $(a \vee b)' = a' \wedge b'$ (ii) $(a \wedge b)' = a' \vee b'$

(b) In a Boolean algebra prove the following
(i) $a.b + b.c + c.a = (a + b).(b + c).(c + a)$ (ii) $[\{(a'.b')' + c\} + (a' + c')]' = 0$

5. (a) State Boole's expansion theorem. Find complete disjunctive normal form in three variable and show that its value is 1.
(b) Write the following functions into conjunctive normal form, in which maximum number of variables are used.
$$(x + y + z)(xy + x'.z)'$$

OR

(a) Define NAND-gate and NOR-gate. Draw the logic circuit for the following Boolean expressions:-
$$d.(a.b' + a'.c)$$
(b) Draw the Karnaugh map and simplify the following Boolean expressions:-
(i) $AB + AB' + A'B'$ (ii) $ABC + A'BC' + ABC' + A'BC$

6. (a) Draw the Hasse Diagram of the
(i) Set D_{30} of positive integral divisors of 30 with the relation "I"
(ii) Subsets of $S = \{a, b, c\}$ with the inclusion relation $\sqsubseteq$
(b) Show that dual of a lattice is a lattice.

OR

(a) Define bounded lattice.
Let $L\{a_1, a_2, \ldots a_n\}$ be a finite lattice then prove that L is bounded.
(b) A lattice L is distributive if and only if
$$(a \vee b) \wedge (b \vee c) \wedge (c \vee a) = (a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \quad \forall a, b, c \in L$$

7. (a) Define direct product of semigroups. Prove that direct product of any two semigroups is a semigoup.
(b) Prove that every finite semigroup has an idempotent element.

OR

(a) If $(M, *)$ is a commutative monoid than prove that the set of all idempotent elements of M forms a submonoid.
(b) Let $(M, *)$ be a monoid with identity element e and $(T, 0)$ be any algebraic structure (0 is binary operation on T). If a mapping $f: M \longrightarrow T$ is onto and satisfies
$$f(a * b) = f(a) \circ f(b), \ \forall a, b \in M$$
The show that $(T, 0)$ is a monoid with $f(e)$ as its identity element.